

# Online Supplemental Appendix for: Adverse Selection in the Annuity Market and the Role for Social Security

Roozbeh Hosseini  
Arizona State University

## **Abstract**

In this Online Appendix I

1. Describe some equilibrium properties of the model using a two period example.
2. Clarify the difference between full information, first best and ex ante contracting. I also, show that first best cannot be implement by using transaction tax on annuities.
3. Solve the model with exclusive contracts. I show that if contracts are exclusive, the welfare gains are large.
4. Calibrate the model with different assumptions on treatment of defined benefit pension income. I calibrate the model under three alternative assumptions and report welfare calculations for each assumption.
5. Present welfare calculations for alternative value of discount factor.
6. Present welfare calculations for various degrees of heterogeneity in mortality index.
7. Present welfare calculations for alternative utility function for bequest.
8. Report tables in Section 4.2 of the paper by age and marital status.

## A Two-period Example

This section discusses some properties of equilibrium prices and To gain insights about some properties of equilibrium prices and allocations, I study a two-period example.

The economy lasts for two periods. All individuals live through the first period. They are alive in the second period with probability  $P(\theta)$ .  $\theta$  is a non-negative number, has distribution  $G(\cdot)$  (with density  $g(\cdot)$ ), and indexes individuals' frailty.  $P(\cdot)$  is a decreasing function of  $\theta$ . Individuals enjoy consumption while they are alive and leave bequests when they die. Assume that there is no discounting and return on saving is one.

The timing is the following: 1) At the beginning of period 0 before any decision is made, individuals learn their  $\theta$  (therefore, they know the probability that they will be alive in the second period,  $P(\theta)$ ); 2) They make decisions about consumption, savings (which they leave as bequests if the die) and annuities. The consumer problem is

$$\max u(c_0) + (1 - P(\theta))v(k_1) + P(\theta) (u(c_1) + v(k_2))$$

subject to

$$c_0 + k_1 + qa \leq w(1 - \tau)$$

$$c_1 + k_2 \leq k_1 + a + z.$$

The goal is to establish the following results: 1) Household decisions over purchase of annuity are monotone in their type. Individuals with higher  $P(\theta)$  (higher probability of survival) purchase more annuity. 2) Equilibrium prices are unfair (they are above average actuarially fair prices). The first proposition establishes these results. After these results are established, I show that increasing the social security tax increases the equilibrium price of annuity.

**Proposition 1** *Annuity purchase for each mortality type,  $a(\theta; q)$ , is a monotone decreasing function of  $\theta$  (index of mortality). Furthermore, equilibrium price  $q^*$  is higher than the average survival risk in the economy,  $q^* > \int P(\theta)dG(\theta)$ .*

**Proof.** Let's re-write the individual problem

$$\max u(c_0) + (1 - P(\theta))v(k_1) + P(\theta) (u(c_1) + v(k_2))$$

subject to

$$c_0 + k_1 + qa \leq w(1 - \tau)$$

$$c_1 + k_2 \leq k_1 + a + z.$$

To simplify the problem define

$$U(x) = \max_{c_1, k_2} u(c_1) + v(k_2)$$

subject to

$$c_1 + k_2 \leq x$$

Now the consumer problem can be written as

$$\max u(c_0) + (1 - P(\theta))v(k_1) + P(\theta)U(k_1 + a + z)$$

subject to

$$c_0 + k_1 + qa \leq w(1 - \tau)$$

Let  $c_0(\theta)$ ,  $k_1(\theta)$  and  $a(\theta)$  be the solution for type  $\theta$ .<sup>1</sup> The first order conditions are

$$u'(c_0(\theta)) = P(\theta)U'(z + k_1(\theta) + a(\theta)) + (1 - P(\theta))v'(k_1(\theta)) \quad (1)$$

and

$$qu'(c_0(\theta)) \geq P(\theta)U(z + k_1(\theta) + a(\theta)) \text{ with "=" if } a > 0. \quad (2)$$

**Claim:** Suppose  $\tilde{\theta} < \theta$ ,  $a(\theta) > 0$  and  $a(\tilde{\theta}) > 0$ , then  $a(\tilde{\theta}) > a(\theta)$ .

Note that in order for annuity demand to be positive for at least some  $\theta$  types, we must have  $q < 1$ . Also, note that if  $\tilde{\theta} < \theta$ , then  $P(\tilde{\theta}) > P(\theta)$  ( $\theta$  is an index of mortality). Replace for  $c_0 = w(1 - \tau) - k_1 - qa$  in (2)

$$\frac{qu'(w(1 - \tau) - k_1(\theta) - qa(\theta))}{U'(z + k_1(\theta) + a(\theta))} = P(\theta) < P(\tilde{\theta}) = \frac{qu'(w(1 - \tau) - k_1(\tilde{\theta}) - qa(\tilde{\theta}))}{U'(z + k_1(\tilde{\theta}) + a(\tilde{\theta}))} \quad (3)$$

Now add (2) and (1)

$$(1 - q)u'(c_0(\theta)) = (1 - P(\theta))v'(k_1(\theta))$$

and replace for  $c_0$  again

$$\frac{(1 - q)u'(w(1 - \tau) - k_1(\theta) - qa(\theta))}{v'(k_1(\theta))} = 1 - P(\theta) > 1 - P(\tilde{\theta}) = \frac{(1 - q)u'(w(1 - \tau) - k_1(\tilde{\theta}) - qa(\tilde{\theta}))}{v'(k_1(\tilde{\theta}))} \quad (4)$$

Suppose  $k_1(\theta) + qa(\theta) \geq k_1(\tilde{\theta}) + qa(\tilde{\theta})$ . Then in order for inequality (3) to hold it must

---

<sup>1</sup>Allocations depend on  $q$  too. I ignore this for now to simplify notation.

be true that  $k_1(\theta) + a(\theta) < k_1(\tilde{\theta}) + a(\tilde{\theta})$ . This implies  $a(\tilde{\theta}) > a(\theta)$ . On the other hand if  $k_1(\theta) + qa(\theta) < k_1(\tilde{\theta}) + qa(\tilde{\theta})$ , then inequality (4) implies that  $k_1(\theta) > k_1(\tilde{\theta})$  which in turn implies that  $a(\tilde{\theta}) > a(\theta)$ .

This implies that there exists a cut-off  $\theta_c$  such that  $a(\theta) > 0$  for all  $\theta < \theta_c$  and  $a(\theta) = 0$  for all  $\theta > \theta_c$ .

Next we show that the equilibrium price is higher than average risk in the economy. Note that if no individual buys annuity, then by equilibrium definition insurers have the most pessimistic belief about buyers, and the equilibrium price is

$$q^* = \max_{\theta} P(\theta) > \int P(\theta)dG(\theta).$$

Assume a positive mass purchase annuity so that aggregate annuity purchase is  $\int a(\theta)dG(\theta) > 0$ . Then the zero profit condition implies that

$$q^* \int_{\underline{\theta}}^{\bar{\theta}} a(\theta)dG(\theta) - P(\theta) \int_{\underline{\theta}}^{\bar{\theta}} a(\theta)dG(\theta) = 0$$

If  $q^* > P(\theta)$  for all  $\theta \leq \theta_c$ , the claim of the proposition is established. Also,  $q^* < P(\theta)$  for all  $\theta \leq \theta_c$  cannot be an equilibrium since at such a price there is demand for annuity but the insurer will not supply annuity. Therefore, there must exist a type  $\theta^* < \theta_c$  such that  $q < P(\theta)$  for  $\theta < \theta^*$  and  $q^* > P(\theta)$  for  $\theta > \theta^*$ .

We can write the insurer's profit as

$$\begin{aligned} 0 &= \int_{\underline{\theta}}^{\bar{\theta}} (q^* - P(\theta))a(\theta)dG(\theta) \\ &= \int_{\underline{\theta}}^{\theta^*} (q^* - P(\theta))a(\theta)dG(\theta) + \int_{\theta^*}^{\bar{\theta}} (q^* - P(\theta))a(\theta)dG(\theta) \\ &< a(\theta^*) \int_{\underline{\theta}}^{\theta^*} (q^* - P(\theta))dG(\theta) + a(\theta^*) \int_{\theta^*}^{\bar{\theta}} (q^* - P(\theta))dG(\theta) \\ &= a(\theta^*) \int_{\underline{\theta}}^{\bar{\theta}} (q^* - P(\theta))dG(\theta) \end{aligned}$$

since  $\theta^* < \theta_c$ , we know that  $a(\theta^*) > 0$  and therefore

$$q^* > \int_{\underline{\theta}}^{\bar{\theta}} P(\theta)dG(\theta)$$

■

This proposition highlights the effect of adverse selection in increasing the price of insurance above the actuarially fair price in this environment. Individuals with a higher probability of survival demand more annuity insurance at any price. They also survive to the second period with higher probability and therefore are more likely to claim the insurance they have purchased. Any unit of coverage that is sold to these individuals is more risky from the point of view of insurers. On the other hand, individuals with a lower probability of survival are less risky for insurers since they are less likely to survive and claim insurance coverage. However, since they are less likely to survive, they purchase less insurance (relative to high survival types). As a result, the insurers are left with a pool of claims more likely to be materialized than the average probability of survival in the population. The risk in each insurer's pool is higher than what is implied by average risk of survival. Therefore, the equilibrium price of annuity is higher than the actuarially fair value of its payout. This is the essence of adverse selection in this environment.

Next, I show that increasing social security taxes leads to an increase in annuity price.

**Proposition 2** *Suppose  $v(\cdot) = \xi u(\cdot)$  for some constant  $\xi > 0$  and  $u(\cdot)$  is homothetic. Then, if aggregate demand for annuity is positive, equilibrium price in the annuity market is an increasing function of social security tax,  $\tau$ .*

**Proof.** The problem of the consumer is

$$\max u(c_0) + (1 - P(\theta))\xi u(k_1) + P(\theta)U(k_1 + a + z)$$

subject to

$$c_0 + k_1 + qa \leq w(1 - \tau)$$

Given that  $u(\cdot)$  is homothetic and given how  $U(\cdot)$  is defined,  $U(\cdot)$  is also homothetic. Make the following change of variable  $x = k + a + z$  and rewrite the consumer problem

$$\max u(c_0) + (1 - P(\theta))\xi u(k_1) + P(\theta)U(x)$$

subject to

$$c_0 + (1 - q)k_1 + qx \leq w(1 - \tau) + qz.$$

By homotheticity of the objective function we know that (if the solution is interior)

$$\begin{aligned} c_0(\theta; q, \tau) &= \phi_c(\theta, q)(w(1 - \tau) + qz) \\ k_1(\theta; q, \tau) &= \phi_k(\theta, q)(w(1 - \tau) + qz) \\ x(\theta; q, \tau) &= \phi_x(\theta, q)(w(1 - \tau) + qz) \end{aligned}$$

for some functions  $\phi_c(\theta, q)$ ,  $\phi_k(\theta, q)$  and  $\phi_x(\theta, q)$ . Then

$$a(\theta; q, \tau) = \max[\phi_a(\theta, q)(w(1 - \tau) + qz) - z, 0]$$

for some function  $\phi_a(\theta, q) = \phi_x(\theta, q) - \phi_k(\theta, q)$ . Note that since  $a(\theta; q, \tau)$  is a monotone decreasing function of  $\theta$  we know that  $\phi_a(\theta, q)$  is positive and monotone decreasing in  $\theta$ .

Replace for  $z = \frac{\tau w}{\bar{P}}$  where  $\bar{P} = \int P(\theta)dG(\theta)$

$$a(\theta; q, \tau) = \max[\phi_a(\theta, q)(w(1 - \tau) + \frac{q}{\bar{P}}w\tau) - \frac{w\tau}{\bar{P}}, 0].$$

Note that  $a(\theta; q, \tau)$  is differentiable in  $\tau$ .

**Lemma 1** *Whenever  $a(\theta; q, \tau) > 0$ ,  $\frac{\partial a(\theta; q, \tau)}{\partial \tau} < 0$  and  $\frac{\partial a(\tilde{\theta}; q, \tau)}{\partial \tau} > \frac{\partial a(\theta; q, \tau)}{\partial \tau}$  for any  $\tilde{\theta} < \theta$ .*

**Proof.** First we show that  $q\phi(\theta, q) < 1$ . Note that from the consumer budget constraint we know

$$\phi_c(\theta, q) + (1 - q)\phi_k(\theta, q) + q\phi_x(\theta, q) = 1$$

then

$$\phi_c(\theta, q) + 1\phi_k(\theta, q) + q(\phi_x(\theta, q) - \phi_k(\theta, q)) = 1$$

and therefore

$$q\phi_a(\theta, q) = q(\phi_x(\theta, q) - \phi_k(\theta, q)) < 1.$$

If  $a(\theta; q, \tau) > 0$ ,

$$a(\theta; q, \tau) = \phi_a(\theta, q)w + \phi_a(\theta, q)(\frac{q}{\bar{P}} - 1)w\tau - \frac{w\tau}{\bar{P}}$$

and

$$\frac{\partial a(\theta; q, \tau)}{\partial \tau} = \frac{q\phi_a(\theta, q)}{\bar{P}}w - \frac{1}{\bar{P}}w - \phi_a(\theta, q)w < 0.$$

Now suppose  $\tilde{\theta} < \theta$  and suppose  $a(\theta; q, \tau) > 0$ . Then  $a(\tilde{\theta}; q, \tau) > a(\theta; q, \tau)$  implies that

$\phi_a(\tilde{\theta}, q) > \phi_a(\theta, q)$ . Therefore,

$$\frac{\partial a(\tilde{\theta}; q, \tau)}{\partial \tau} - \frac{\partial a(\theta; q, \tau)}{\partial \tau} = (\phi_a(\tilde{\theta}, q) - \phi_a(\theta, q)) \left( \frac{q}{P} - 1 \right) w \tau > 0$$

■

Next, I proceed to prove the theorem.

Define function  $H(q; \tau)$  as

$$H(q; \tau) = \frac{\int P(\theta) a(\theta; q, \tau) dG(\theta)}{\int a(\theta; q, \tau) dG(\theta)}.$$

The equilibrium price is the lowest fixed point of this function.

$$q^* = \inf_q \{q | H(q; \tau) \leq q\}.$$

I will show that increase in social security tax,  $\tau$ , shifts this function upward and therefore increases the equilibrium price.

First note that for any price  $q$  we have

$$\begin{aligned} H(q; \tau) - \int P(\theta) dG(\theta) &= \frac{\int P(\theta) a(\theta; q, \tau) dG(\theta) - \int a(\theta; q, \tau) dG(\theta) \int P(\theta) dG(\theta)}{\int a(\theta; q, \tau) dG(\theta)} \\ &= \frac{Cov(P(\theta), a(\theta; q, \tau))}{\int a(\theta; q, \tau) dG(\theta)} > 0. \end{aligned}$$

in which the last inequality is true because both  $P(\theta)$  and  $a(\theta; q, \tau)$  are monotone decreasing in  $\theta$ .

Since  $a(\theta; q, \tau)$  is differentiable in  $\tau$ ,  $H(q; \tau)$  is also differentiable in  $\tau$ . For any price  $q$

$$\begin{aligned} \frac{\partial H(q; \tau)}{\partial \tau} &= \frac{\left( \int P(\theta) \frac{\partial a(\theta; q, \tau)}{\partial \tau} dG(\theta) \right) \left( \int a(\theta; q, \tau) dG(\theta) \right) - \left( \int P(\theta) a(\theta; q, \tau) dG(\theta) \right) \left( \int \frac{\partial a(\theta; q, \tau)}{\partial \tau} dG(\theta) \right)}{\left( \int a(\theta; q, \tau) dG(\theta) \right)^2} \\ &= \frac{\int P(\theta) \frac{\partial a(\theta; q, \tau)}{\partial \tau} dG(\theta) - H(q; \tau) \int \frac{\partial a(\theta; q, \tau)}{\partial \tau} dG(\theta)}{\int a(\theta; q, \tau) dG(\theta)} \\ &= \frac{\left( \int (P(\theta) - H(q; \tau)) dG(\theta) \right) \left( \int \frac{\partial a(\theta; q, \tau)}{\partial \tau} dG(\theta) \right) + Cov(P(\theta) - H(q; \tau), \frac{\partial a(\theta; q, \tau)}{\partial \tau})}{\int a(\theta; q, \tau) dG(\theta)} \\ &> 0 \end{aligned}$$

The inequality is true because  $\frac{\partial a(\theta; q, \tau)}{\partial \tau} \leq 0$  for all  $\theta$  and  $H(q; \tau) > \int P(\theta) dG(\theta)$ . Also, as I have shown above  $\frac{\partial a(\theta; q, \tau)}{\partial \tau}$  is monotone decreasing in  $\theta$ . This proves that  $H(q; \tau)$  is always increasing with  $\tau$ . Therefore, its lowest fixed point must also increase as  $\tau$  increases.<sup>2</sup> This completes the proof of the theorem. ■

Social security is a substitute for annuity that is purchased in the market. An increase in social security tax causes everyone to reduce demand for annuity in the market. Such an increase has a larger effect on demand for annuity by lower survival types. The reason is that increasing tax (and benefits) of social security has two effects. On the one hand it substitutes for annuities and therefore reduces demand in the market. This effect is the same for all types. On the other hand it provides annuities at cheaper rates (than is available in the market). This generates an income effect that increases demand. But the magnitude of this income effect depends on probability of survival and is larger for high survival types. The reason is that these types survive with higher probability and are more likely to collect social security benefits. Therefore, the overall reduction in annuity demand is larger for low survival types than for high survival types, and the profile of annuity purchase becomes skewed towards high survival types. As a result, increasing social security increases risk in the annuity pool in the market. This in turn leads to higher equilibrium prices.

## Sensitivity to Risk Aversion Parameter

In this section assume (as I do in the quantitative exercises in the main text) that utility over consumption is CRRA with coefficient  $\gamma$

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

and utility over bequest is

$$v(b) = \xi \frac{b^{1-\gamma}}{1-\gamma}.$$

I will show how equilibrium prices in the annuity market change as we change these parameters.

Suppose there is no demand for bequest ( $\xi = 0$ ) and there is no social security. First-order condition for individual  $\theta$  is<sup>3</sup>

$$q^a c_0^{-\gamma} = P(\theta) a^{-\gamma}$$

---

<sup>2</sup>See [Milgrom and Roberts \(1994\)](#) for detailed arguments.

<sup>3</sup>Since annuity makes survival contingent payment it dominates savings, and therefore I omit the first-order condition with respect to  $k_1$ . For more detailed analysis of this, see [Brown, Davidoff, and Diamond \(2005\)](#).



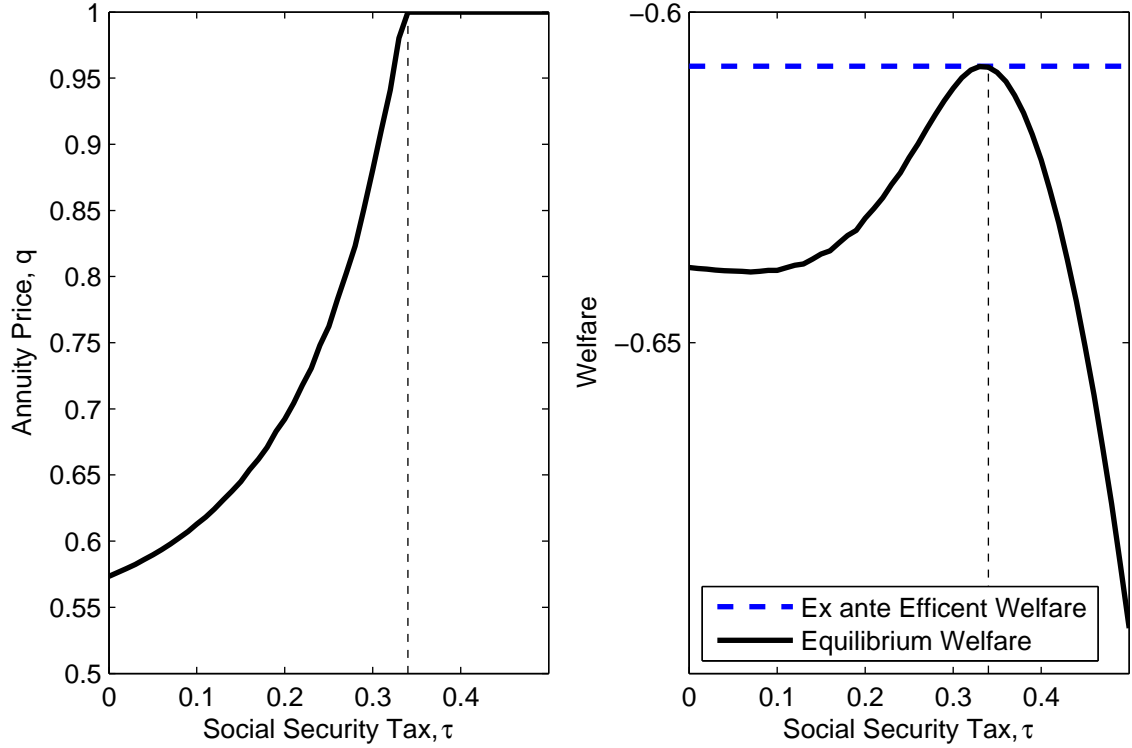


FIGURE 1: Effect of social security tax on equilibrium price and welfare. Left panel shows the equilibrium price of annuity for various levels of social security tax. Right panel shows ex ante welfare for various levels of social security tax.

and the budget constraint

$$c_0 + q^a a = w.$$

Therefore,

$$a(q; P(\theta)) = \frac{w}{q + \left(\frac{q}{P(\theta)}\right)^{\frac{1}{\gamma}}}.$$

Fixing price  $q$  and taking a derivative with respect to  $\gamma$ ,

$$\frac{\partial a(q; P(\theta))}{\partial \gamma} = \frac{w \left(\frac{q}{P(\theta)}\right)^{\frac{1}{\gamma}} \log \left(\frac{q}{P(\theta)}\right)}{\gamma^2 \left(q + \left(\frac{q}{P(\theta)}\right)^{\frac{1}{\gamma}}\right)^2}.$$

Notice that for individuals with  $P(\theta) < q$ , increasing  $\gamma$  leads to more demand for insurance. This is because more risk-averse individuals demand more insurance. However, notice that if  $P(\theta) > q$ , then the effect is reversed. For these individuals, the elasticity of substitution effect dominates. They favor a smoother consumption path across time, and therefore their demand for annuity is reduced (instead they consume more in the first period). Overall,

higher demand by high mortality types ( $P(\theta) < q$ ) and lower demand by low mortality types ( $P(\theta) > q$ ) mean that overall risk in the insurers' pool is lower. That is because there is more demand by good risk types (who are less likely to claim their coverage) and less demand by high risk types (who are more likely to claim their coverage). This leads to a reduction in the equilibrium price.

**Lemma 2** *Equilibrium annuity price is a decreasing function of the coefficient of risk aversion.*

**Proof.** Consider the zero profit condition of insurers:

$$\int (q - P(\theta)a(q; P(\theta)))dG(\theta) = 0.$$

Take the derivative with respect to  $\gamma$ :

$$\int \frac{\partial q}{\partial \gamma} a(q; P(\theta))dG(\theta) + \int (q - P(\theta)) \left[ \frac{\partial q}{\partial \gamma} \frac{\partial a(q; P(\theta))}{\partial q} + \frac{\partial a(q; P(\theta))}{\partial \gamma} \right] dG(\theta) = 0.$$

Collecting terms:

$$\frac{\partial q}{\partial \gamma} = \frac{- \int (q - P(\theta)) \frac{\partial a(q; \theta)}{\partial \gamma} dG(\theta)}{\int [a(q; \theta) + (q - P(\theta)) \frac{\partial a(q; \theta)}{\partial q}] dG(\theta)}.$$

The denominator is always positive since  $\frac{\partial a(q; \theta)}{\partial q}$  and  $P(\theta)$  are both decreasing functions of  $\theta$ . The numerator is always negative since  $\frac{\partial a(q; \theta)}{\partial \gamma}$  is positive whenever  $P(\theta) < q$  and is negative whenever  $P(\theta) > q$ . Therefore, it follows that

$$\frac{\partial q}{\partial \gamma} < 0.$$

■

Figure 2 is a graphical illustration of this result. Panel (a) shows the profile of annuity purchase as a function of probability of survival,  $P(\theta)$ , for a given price. We see that at a higher level of risk aversion, the profile of annuity purchase is less steep. This leads to a less skewed anticipated distribution for payouts (panel (b)) and, in turn, the price is going to be lower.

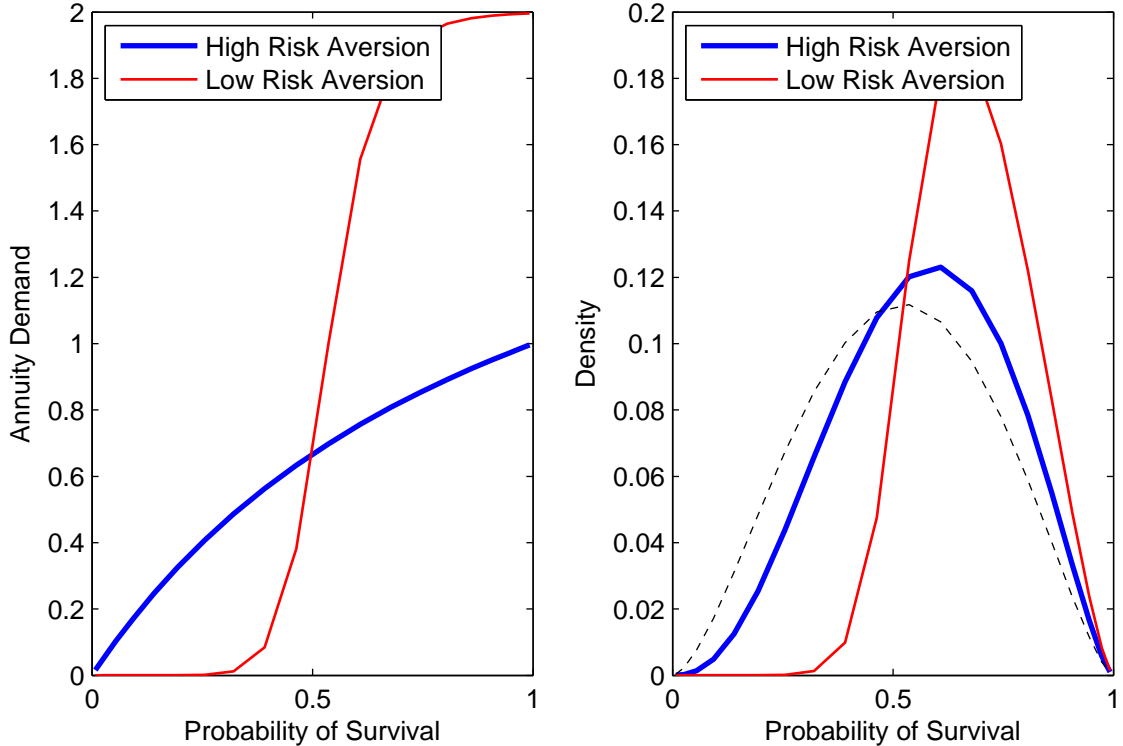


FIGURE 2: Left panel shows the profile of annuity purchase for various survival types at a given price. Right panel shows the anticipated distribution of payouts. The dashed line is the distribution of types.

## B Welfare Comparisons – Few Clarifications

The purpose of this section is to clarify some of the welfare comparison in the main paper. In all the discussions individuals are alive for at most two periods and they are heterogeneous in their survival probability. There are no other heterogeneity. The only shock is death/survival. Let  $\theta \in \Theta$  be the mortality index and  $P(\theta)$  be the probability of survival to second period. Let  $G(\theta)$  be the distribution of  $\theta$ . Assume no discounting and return on saving equal to one. Here, I describe three benchmark allocations:

1. Ex ante efficient allocation (or first best): This is the solution to the utilitarian planner problem that puts the same weight on all mortality type. It maximizes the ex ante welfare of individuals behind the veil ignorance (before their mortality/survival type is realized). I call this benchmark FB for first best.

Planning problem is:

$$\max_{C_1(\theta), C_2(\theta)} \int_{\theta \in \Theta} [u(C_1(\theta)) + P(\theta) u(C_2(\theta))] dG(\theta)$$

s.t.

$$\int_{\theta \in \Theta} [C_1(\theta) + P(\theta) C_2(\theta)] dG(\theta) = w$$

Note that the first order condition in the problem implies

$$\begin{aligned} C_1(\theta) &= C_1^{FB} \quad \text{for all } \theta \\ C_2(\theta) &= C_2^{FB} \quad \text{for all } \theta \\ C_1^{FB} &= C_2^{FB} = \frac{w}{1 + \int_{\theta \in \Theta} P(\theta) dG(\theta)} \end{aligned}$$

Therefore, the allocations are independent of types. Consumption allocations are age-independent here because of the simplifying assumption that discount factor is equal to the inverse of return on saving (and both are assumed to be equal 1).

**Claim 1** *First best allocations are incentive compatible.*

**Proof.** Since everyone receives the same allocation there is no benefit from misreporting mortality type. Therefore, allocations are incentive compatible. ■

**Remark 1:** The expected utility assumption is important for this result. If, for example, the utility is recursive, the allocation depend on mortality type and will no longer be incentive compatible.

**Remark 2:** If there are other heterogeneities in the model (i.e., in preferences, income, etc). Then the first best allocation will not be incentive compatible.

**Remark 3:** In this environment individuals face two shocks: 1) the realization of their mortality/survival type; 2) the realization of their death/survival. FB allocation provides perfect insurance against both shocks.

2. A decentralized economy in which individuals buy annuity contracts at actuarially fair price after they realize their mortality/survival types. Since there is heterogeneity in survival probabilities each person is facing a different price. I call this allocation FI for full information.

Individual of type  $\theta$  face the annuity price  $q(\theta)$  and solve the following problem:

$$(C_1^{FI}(\theta), C_2^{FI}(\theta)) = \arg \max_{C_1, C_2} u(C_1) + P(\theta) u(C_2)$$

s.t.

$$\begin{aligned} C_2 + q(\theta) a &= w \\ C_2 &= a \end{aligned}$$

The first order condition is

$$q(\theta) u'(C_1^{FI}(\theta)) = P(\theta) u'(C_2^{FI}(\theta))$$

If there is full informant (and no other frictions), the  $q(\theta) = P(\theta)$ . In other words, each individual pays an actuarially fair price of the annuity that s/he buys. Imposing equilibrium price, we get

$$C_1^{FI}(\theta) = C_2^{FI}(\theta) = \frac{w}{1 + P(\theta)}.$$

Therefore, FI allocations are no first best. Although, these allocations are still pareto efficient. Under full information, the first welfare theorem holds. But they are not ex ante efficient. FI allocations provide full insurance against survival risk conditional on mortality/survival type. However, from ex ante point of view individuals face another shock. That is, the realization of their mortality type. That shock is not insured under FI set up discussed above.

3. A decentralized economy in which individuals buy annuity contracts before the realization of their mortality/survival types. Since at the time of the purchase everyone is identical, everyone face the same price for the annuity. This is the Prescott-Townsend world. I call this allocation PT for Prescott-Townsend.

Suppose individuals have to trade before period 1. Note that since they make purchase before they know their types, neither consumption allocation nor annuity purchase can be contingent on mortality type. Everyone must pay price  $q$  for each unit of annuity coverage. Individuals solve the following problem:

$$(C_1^{PT}, C_2^{PT}) = \arg \max_{C_1, C_2} \int_{\theta \in \Theta} [u(C_1) + P(\theta) u(C_2)] dG(\theta)$$

s.t.

$$\begin{aligned} C_1 + qa &= w && \text{for all } \theta \\ C_2 &= a \end{aligned}$$

The first order condition is

$$qu'(C_1^{PT}) = u'(C_2^{PT}) \int_{\theta \in \Theta} P(\theta) dG(\theta)$$

Since there is no other friction, insurers must sell the annuity at the population average survival risk

$$q = \int_{\theta \in \Theta} P(\theta) dG(\theta).$$

Therefore,

$$C_1^{PT} = C_2^{PT} = \frac{w}{1 + \int_{\theta \in \Theta} P(\theta) dG(\theta)}$$

It immediately follows that

$$\begin{aligned} C_1^{PT} &= C_1^{FB} \\ C_2^{PT} &= C_2^{FB} \end{aligned}$$

Therefore, the ex ante efficient allocation (first best) is identical to decentralized allocation if individuals trade before the realization of their mortality types and are not allowed to trade afterwards. Note that the PT allocation is not immune to trade after realization of type. High mortality individuals would like to sell their annuities and low mortality individuals would like to buy. This, of course, will lead to adverse selection problem if mortality/survival types are private information.

## Implementation of first best allocation

Can the first best be implemented by tax on transaction (and lump sum transfer)?

Suppose mortality/survival type is private information. Suppose price of annuity is  $q$ . Note that absence of annuity market can be modeled as  $q = 1$ . Individuals must pay a tax  $T(a)$  if they purchase annuity coverage  $a$ . Let  $Z_1$  and  $Z_2$  be lump sum tax/benefits

$$\max_{C_1, C_2} u(C_1) + P(\theta) u(C_2)$$

s.t.

$$\begin{aligned} C_1 + qa &= w + Z_1 \\ C_2 &= a - T(a) + Z_2 \end{aligned}$$

The first order condition

$$qu'(C_1(\theta)) = P(\theta)u'(C_2(\theta))(1 - T'(a))$$

The question we are after is what should the tax function look like in order to implement first best allocations. Recall that first best allocations are independent of type  $\theta$

$$1 - T'(a) = \frac{qu'(C_1^{FB})}{P(\theta)u'(C_2^{FB})}$$

Note that the right hand side depends on mortality type (through  $P(\theta)$ ). In order to implement this allocation using the proposed tax on annuities, marginal tax must depend on survival probability. But that is not observed. Therefore, it is not possible to implement type independent allocations with tax on annuity purchase.

A policy than can implement first best allocation is the lump sum tax and transfer discussed in the paper (other than PT set up discussed above):

$$\begin{aligned} Z_1^* + Z_2^* \int_{\theta \in \Theta} P(\theta) dG(\theta) &= 0 \\ Z_2^* &= C_2^{FB} = \frac{w}{1 + \int_{\theta \in \Theta} P(\theta) dG(\theta)} \\ Z_1^* &= -C_2^{FB} \int_{\theta \in \Theta} P(\theta) dG(\theta) = -\frac{w \int_{\theta \in \Theta} P(\theta) dG(\theta)}{1 + \int_{\theta \in \Theta} P(\theta) dG(\theta)} \end{aligned}$$

With this policy, the demand for annuity is zero for all type and  $q = \max_{\theta} P(\theta)$ .

Suppose not, i.e., suppose  $q < \max_{\theta} P(\theta)$ . Then, there is a type  $\tilde{\theta}$  such that  $P(\tilde{\theta}) = q$ . Then, for this type the first order conditions hold with  $a = 0$ . And consumption equal to

$$\begin{aligned} C_1 &= w + Z_1^* \\ C_2 &= Z_1^* \end{aligned}$$

Therefore, for all types such that  $P(\theta) < P(\tilde{\theta})$  the demand for annuity must be zero. This implies that the equilibrium price of annuity must be larger than  $P(\tilde{\theta})$  (since there are higher survival risk types who purchase positive annuity). This is a contradiction. Therefore, the equilibrium price must be equal to  $\max_{\theta} P(\theta)$  and demand for annuity must be zero for all types.

## C Exclusive Contracts

In this section I show that the quantitative conclusions of the paper will be very different if annuity contracts are assumed to be exclusive. Insurance markets with adverse selection and exclusive contracts are difficult to analyze. There is a long literature starting with [Rothschild and Stiglitz \(1976\)](#) that explores existence, efficiency and uniqueness properties of equilibria with adverse selection and exclusive contracts.<sup>4</sup>

Here, I assume Rothschild-Stiglitz type equilibrium exists and I solve for the equilibrium allocation. The main goal of this section is to show that welfare calculations of the paper are sensitive to the (non)exclusivity assumption. The argument is that, *if* annuity contracts are exclusive and *if* equilibrium exists, *then* the welfare gains from mandatory annuitization are large. However, as I argue in the paper there is little evidence to support exclusivity of annuity contracts.<sup>5</sup>

### Annuity market with exclusive contracts

A contract is an annuity premium  $q^{RS}(\theta) a(\theta)$  and coverage  $a(\theta)$ . Here  $q^{RS}(\theta)$  is the implicit unit annuity price for annuity coverage  $a(\theta)$ . Consider the problem of individual type  $\theta$  who chooses the contract  $(q(\theta') a(\theta'), a(\theta'))$ . This individual solves the following problem

$$U(\theta, \theta') = \max \sum_{t=0}^{\infty} \beta^t \left[ P_t(\theta) \frac{c_t^{1-\sigma}}{1-\sigma} + (P_t(\theta) - P_{t+1}(\theta)) \beta \xi \frac{b_{t+1}^{1-\sigma}}{1-\sigma} \right]$$

s.t.

$$c_t + k_{t+1} = Rk_t + w(1 - \tau) \quad \text{for } t < J$$

$$c_J + k_{J+1} - q^{RS}(\theta') a(\theta') = Rk_J + w(1 - \tau)$$

$$c_t + k_{t+1} = Rk_t + z + a(\theta') \quad \text{for } t > J$$

Let  $U_0$  be the utility of an individual who chooses not to purchase any annuity

$$U_0(\theta) = \max \sum_{t=0}^{\infty} \beta^t \left[ P_t(\theta) \frac{c_t^{1-\sigma}}{1-\sigma} + (P_t(\theta) - P_{t+1}(\theta)) \beta \xi \frac{b_{t+1}^{1-\sigma}}{1-\sigma} \right]$$

---

<sup>4</sup>See [Dionne, Doherty, and Fombaron \(2000\)](#) for an excellent survey of this literature. For more recent development in the literature see [Netzer and Scheuer \(forthcoming\)](#) and references therein.

<sup>5</sup>See [Finkelstein and Poterba \(2004\)](#) for direct test on convexity of prices and/or bulk discount in the U.K. annuity market. Also, [Cannon and Tonks \(2008\)](#) provide direct evidence that, except at very low coverage, annuity prices do not vary with coverage.



s.t.

$$c_t + k_{t+1} = Rk_t + w(1 - \tau) \quad \text{for } t \leq J$$

$$c_t + k_{t+1} = Rk_t + z \quad \text{for } t > J$$

Let  $q^{AF}(\theta)$  be the actuarially fair price of the unit of annuity coverage for type  $\theta$ . Rothschild and Stiglitz (1976) and Bisin and Gottardi (2006) show that in this environment  $q^{RS}(\theta) = q^{AF}(\theta)$ . Insurers maximize their profit by choosing the coverage  $a(\theta)$  subject to participation and incentive constraints. Let  $g_J(\theta)$  be the distribution of types at age  $J$ . (NOTE: from now on, I assume types are discrete and only upward local constraints bind, i.e., only low mortality types try to lie to be high mortality types). Let  $\tilde{a} = \{a(\theta_i)_{i=1, \dots, n}\}$ . The insurer's problem is

$$\max_{a(\theta_i)_{i=1, \dots, n}} \sum_{i=1}^n q^{AF}(\theta_i) a(\theta_i) g_J(\theta_i)$$

s.t.

$$U(\theta_i, \theta_i; \tilde{a}) \geq U(\theta_i, \theta_{i+1}; \tilde{a}) \quad \text{for all } \theta_i, \theta_{i+1}$$

$$U(\theta_i, \theta_i; \tilde{a}) \geq U_0(\theta_i) \quad \text{for all } \theta_i$$

The equilibrium is a set of separating contracts  $(q(\theta) a(\theta), a(\theta))$  that solves the insurer optimization problem above such that the insurers make non-positive profit.

In this equilibrium the allocation for type  $\theta_1$  (lowest mortality types) is unconstrained. To solve for the equilibrium allocation I choose  $a(\theta_1)$  by solving the maximization problem of type  $\theta_1$  who face the unit annuity price of  $q^{AF}(\theta_1)$  (actuarial fair price of their type). Next, I choose  $a(\theta_{i+1})$  such that

$$U(\theta_i, \theta_i) = U(\theta_i, \theta_{i+1})$$

## Welfare gain from mandatory annuitization

I next solve the model under the assumption of exclusive contracts for benchmark parametrization and report ex ante welfare gains from current US social security system over an economy with exclusive contracts and without social security.

Table 1 shows the ex ante welfare calculations for equilibrium with exclusive contracts. It is clear that in this case welfare gains from current US social security are large. It is also clear that ex ante cost of private information are large. The reason is that in this environment, with exclusive contracts, there are no cross subsidization across risk types. Therefore, there is no crowding out effect from social security and no effect on prices.

In this equilibrium, in the absence of social security, highest risk types receive as much

insurance as they desire at the actuarially fair price of their survival risk (which is high). However, for all other risk types the insurance coverage is rationed. This is needed to keep the menu of contracts incentive compatible. For a big portion of population this is much lower coverage they would otherwise receive under non-exclusive contracts considered in the paper. That is why from ex ante point of view this equilibrium is much worse than the equilibrium considered in the paper in the absence of social security.

On the other hand, social security is the only instrument to pool risk across morality types when contracts are exclusive. This risk pooling is desirable from ex ante perspective.

To sum up, equilibrium with exclusive contracts is very similar to autarky (at least for the benchmark parameters). In fact, on average only 1.44 percent of retirement wealth is in the form annuities when there is social security (current US replacement ratio). This rises to about 12 percent without social security (compare this to 10 percent and 67 percent in model with non-exclusive contracts).

Figure 3 plots expected utility of each mortality type under assumption of exclusive and non-exclusive contracts. As we see, when there is no social security the outcome is much worse for those in the low mortality (high risk of survival). If contracts are non-exclusive they receive significant cross-subsidization from higher mortality (better risk types). There is no such cross subsidy when contracts are exclusive and there is no social security.

TABLE 1: Welfare Calculations: Exclusive Contracts vs Non-exclusive Contracts

	Exclusive Contracts	Non-exclusive Contracts
	Ex ante welfare gains from annuitization in the current US system (%)	
Annuity Market with Private Info.	1.9	0.07
	Welfare losses due to private information (%)	
Without Social Security	2.08	0.38
With Social Security	0.18	0.32
	Welfare gains from having access to annuity market (%)	
Without Social Security	1.70	2.79
With Social Security	0.3	0.17

*Note:* All calculations are done using benchmark parameters reported in Table 4 of the paper.

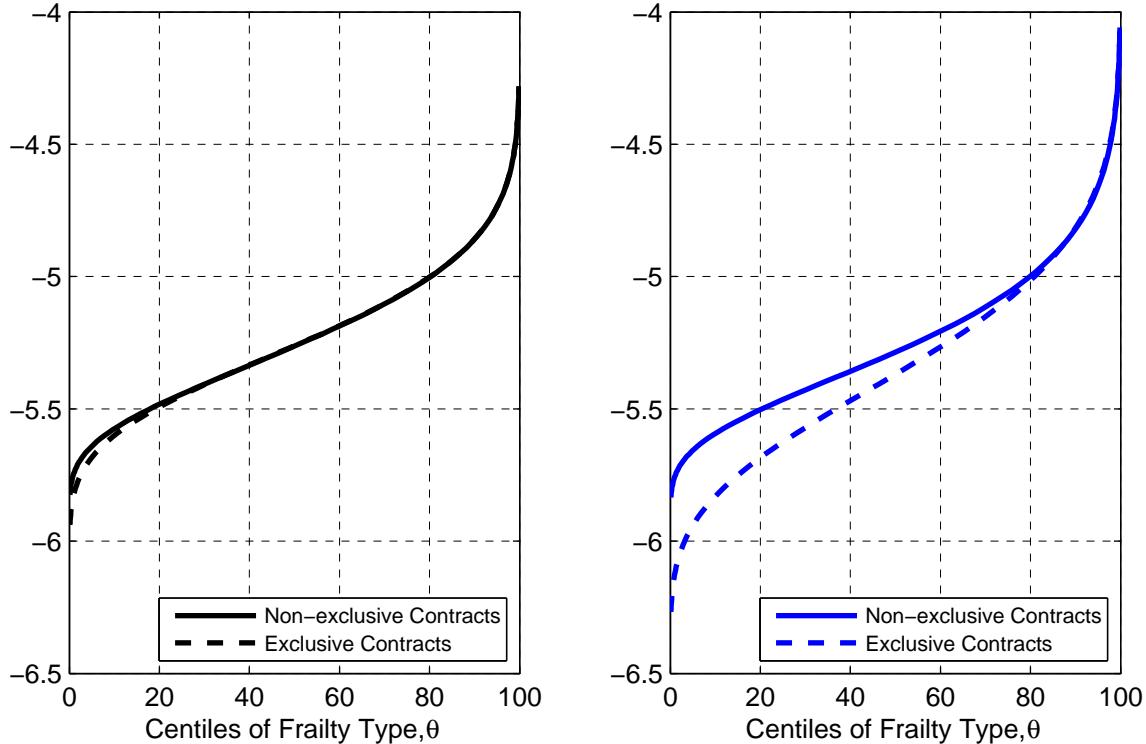


FIGURE 3: Comparing individuals' expected utility under exclusive contracts and non-exclusive contracts. Left panel shows Expected utility under current US system. Right panel shows expected utility without social security.

## D Alternative Treatment of Defined Benefit Pension Income

In the paper I assume holding defined benefit pension plan is similar to purchasing annuity in the market (see Section 5.2 for the discussion). Defined benefit pension is an asset that has to be accounted for when I take model to the data. Here I explore three alternative calibration assumptions.

1. **Alternative Calibration 1:** Assume these assets as non-annuity. Then the fraction of wealth that is annuitized can only come from narrowly defined annuities (private annuities). That fraction is 0.5% and only about 5% of sample hold those assets. In this scenario I calibrate the model to match the 0.5% target for fraction of annuitized wealth.
2. **Alternative Calibration 2:** Assume these assets are mandatory annuities and everyone holds them and there is no choice over their amount. Once again, under this

assumption, I calibrate the model to match the 0.5% target for fraction of annuitized wealth (through private annuities).

- 3. Alternative Calibration 3:** Exclude individuals who only hold defined benefit pension plan from the sample. This results in dropping close to half of the sample. I calculate fraction of wealth that is annuitized and fraction of population who hold annuities in this new sample and I use these values as target for calibration.

Before, I present calibration and welfare calculation results, I present the annuitization data which excludes defined benefit pension recipients from the sample. In this sample I still include those who have annuity and defined benefit pension. But exclude individuals who only receive defined benefit pensions and no other annuity income (other than social security). The results are in Table 2 and 3.

In the restricted sample around 10 percent of individuals holds private annuities. About 60 percent of those who hold private annuities also receive defined benefit pension income (around 6 percent of total sample). On average 1 percent of retirement wealth is held in the form of private annuities and about 0.9 percent in the form defined benefit pension plans. Overall, on average 2 percent of retirement wealth is in the form of private annuities (or defined benefit pension plan). This is going to my target for calibration in ‘Alternative Calibration 3’.

The calibration results are presented in Table 4. The ‘Alternative Calibration 1’ refers to the case in which defined benefit pension annuity is ignored. The ‘Alternative Calibration 2’ refers to the case in which there is no choice in receiving defined benefit income (everyone received the benefit). For this case I introduce a defined benefit replacement ratio and calibrate it so that average ratio of the pension wealth to total retirement is 9.5 percent. ‘Alternative Calibration 3’ refers to the case where defined benefit pension is excluded from the sample and from the calibration. The welfare calculations are presented in Table 5.

In all the alternative calibrations the bequest parameter needed to match the observations are large. This results to negative welfare gains from mandatory annuitization, even in a full information economy.

TABLE 2: Average Fraction of Retirement Wealth that is Annuitized

	Employer Sponsored Pensions (%)	Private Annuities (%)	Social Security (%)
age>60	0.9	1.0	41.0

*Source:* Authors calculations using wave 7 of RAND HRS data, version M.

*Notes:* All calculations are for male respondents who are older than 60 years old and are collecting social security retirement benefit in wave 7 and do not receive a defined benefit pension income (unless they also purchase private annuity).

TABLE 3: Fraction of Population with Private Annuity or Defined Benefit Pension

	In their own name (%)			In own or spouse's name (%)		
	Emp.-spons. Pension	Private Annuity	Pension or Annuity	Emp.-spons. Pension	Private Annuity	Pension or Annuity
age>60	6.12	9.22	10.55	7.30	11.81	11.81

*Source:* Authors calculations using wave 7 of RAND HRS data, version L.

*Notes:* All calculations are for male respondents who are older than 60 years old and are collecting social security retirement benefit in wave 7 and do not receive a defined benefit pension income (unless they also purchase private annuity).

TABLE 4: Calibration for Alternative Treatment of Defined Benefit Pension Income

	Benchmark Calibration	Alternative Calibration 1	Alternative Calibration 2	Alternative Calibration 3
$\xi$	0.9	2.52	1.1	1.91
DB pension replacement ratio	<i>n.a.</i>	<i>n.a.</i>	0.11	<i>n.a.</i>
Fraction with annuity (not targeted)	0.53	0.05	0.06	0.16

*Note* In the first column the bequest parameter is chosen to match the fraction annuitized wealth of 10%. This target for column 2 and 3 is 0.5%. The target for column 4 is 2%.

TABLE 5: Welfare Calculations for Alternative Treatment of Defined Benefit Pension Income

	Benchmark Calibration	Alternative Calibration 1	Alternative Calibration 2	Alternative Calibration 3
Ex ante welfare gains from annuitization in the current US system (%)				
Autarky	2.68	1.15	1.61	1.56
Annuity Market with Full Info.	0.01	-0.58	-0.32	-0.41
Annuity Market with Private Info.	0.07	-0.29	-0.06	-0.20
Annuity Market with Private Info. (with annuity prices fixed)	0.42	-0.07	0.14	0.07
Welfare losses due to private information (%)				
Without Social Security	0.38	0.49	0.42	0.45
With Social Security	0.32	0.19	0.16	0.24
Welfare gains from having access to annuity market (%)				
Without Social Security	2.79	1.44	1.67	1.79
With Social Security	0.17	0.00	0.00	0.02
Ex ante welfare gains from implementing the first best (%)				
Autarky	4.32	3.59	3.22	3.69
Annuity Market with Full Info.	1.11	1.61	1.11	1.41
Annuity Market with Private Info.	1.49	2.10	1.53	1.87

## E Sensitivity to discount factor

In the paper I assume discount factor of  $\beta = 0.97$  and 3 percent real return on saving. These are the middle range of the values used in the life-cycle and annuity literature. However, since present values are sensitive to discounting, the welfare calculations can be sensitive, too. Here I report calibration and welfare calculations assuming discount factor of  $\beta = 0.98$  and  $\beta = 0.96$  (with real return in saving 2 and 4 percent respectively).

The calibration results are presented in Table 6. Welfare calculations are presented in Table 7.

TABLE 6: Calibrated Bequest Parameter for Different Discount Factor

Calibrated weight on bequest			
(Chosen to match average annuitized wealth of 10% at age 65–70)			
	$\beta = 0.96$	$\beta = 0.97^*$	$\beta = 0.98$
$\xi$	0.94	0.9	0.8
Fraction with annuity (not targeted)	0.53	0.53	0.53

*Note* For each value of risk aversion parameter,  $\xi$  is chosen to such that average fraction of annuitized wealth at retirement is 10%. All other parameters are the same as benchmark (Table 4 in the paper.)

\* Benchmark calibration.

TABLE 7: Welfare Calculations for Different Discount Factor

	$\beta = 0.96$	$\beta = 0.97^*$	$\beta = 0.98$
Ex ante welfare gains from annuitization in the current US system (%)			
Autarky	1.42	2.68	4.76
Annuity Market with Full Info.	0.25	0.01	0.45
Annuity Market with Private Info.	-0.20	0.07	0.50
Annuity Market with Private Info. (with annuity prices fixed)	0.03	0.42	1.02
Welfare losses due to private information (%)			
Without Social Security	0.26	0.38	0.50
With Social Security	0.21	0.32	0.45
Welfare gains from having access to annuity market (%)			
Without Social Security	1.73	2.79	4.51
With Social Security	0.10	0.17	0.25
Ex ante welfare gains from implementing the first best (%)			
Autarky	3.20	4.32	6.36
Annuity Market with Full Info.	1.19	1.11	1.26
Annuity Market with Private Info.	1.45	1.49	1.78

\* Benchmark calibration.

## F Sensitivity to heterogeneity in mortality

In the paper I use standard deviation of 0.56 for the initial distribution of  $\log(\theta)$ . In this section I report calibration and welfare calculations for two alternative values to check sensitivity of the results. I use  $\sigma_\theta = 0.28$  as low benchmark and  $\sigma_\theta = 0.7$  as a high benchmark for heterogeneity in mortality index. In each case I calibrate the bequest parameter  $\xi$  so that the average fraction of retirement wealth that is annuitized is equal to 10%.

The calibration results are presented in Table 8. Welfare calculations are presented in Table 9. The results are not sensitive to the heterogeneity in mortality within the range that I consider.

TABLE 8: Calibrated Bequest Parameter for Different Heterogeneity in Mortality Distribution

Calibrated weight on bequest (Chosen to match average annuitized wealth of 10% at age 65–70)			
	$\sigma_\theta = 0.28$	$\sigma_\theta = 0.56^*$	$\sigma_\theta = 0.70$
$\xi$	0.88	0.9	0.71
Fraction with annuity (not targeted)	0.53	0.53	0.62

*Note* For each value of risk aversion parameter,  $\xi$  is chosen to such that average fraction of annuitized wealth at retirement is 10%. All other parameters are the same as benchmark (Table 4 in the paper.)

\* Benchmark calibration.



TABLE 9: Welfare Calculations for Different Heterogeneity in Mortality Distribution

	$\sigma_\theta = 0.28$	$\sigma_\theta = 0.56^*$	$\sigma_\theta = 0.70$
Ex ante welfare gains from annuitization in the current US system (%)			
Autarky	2.98	2.68	2.87
Annuity Market with Full Info.	0.05	0.01	0.10
Annuity Market with Private Info.	0.03	0.07	0.08
Annuity Market with Private Info. (with annuity prices fixed)	0.42	0.42	0.45
Welfare losses due to private information (%)			
Without Social Security	0.31	0.38	0.30
With Social Security	0.33	0.32	0.32
Welfare gains from having access to annuity market (%)			
Without Social Security	3.14	2.79	3.01
With Social Security	0.19	0.17	0.22
Ex ante welfare gains from implementing the first best (%)			
Autarky	4.71	4.32	4.37
Annuity Market with Full Info.	1.21	1.11	1.02
Annuity Market with Private Info.	1.52	1.49	1.32

\* Benchmark calibration.

## G Non-homothetic Preferences

In this section I report results of welfare calculations for an alternative utility for bequest which is of the following form:

$$v(b) = \xi \frac{(\varpi + b)^{1-\gamma}}{1-\gamma}$$

This utility function is used in many papers that study intergenerational transfers and effect of bequest motives on asset accumulation and annuitization.<sup>6</sup>

The calculations in the paper is done for the case  $\varpi = 0$ . For positive  $\varpi$ , the bequest is luxury good. Therefore, it is desired more by higher income individuals. On the other hand low income individuals have less desire to leave bequest. In my model lower mortality

<sup>6</sup>See for example Ameriks, Caplin, Laufer, and Van Nieuwerburgh (2011), De Nardi (2004), Lockwood (2012) and Pashchenko (2013).

individuals accumulate more assets and therefore have more income. Therefore, they have more desire to leave bequest under this utility parametrization. On the other hand, higher mortality individuals accumulate fewer assets and have less desire to leave bequest (in fact some of them do not leave bequest at all). This is in some sense the opposite of the findings in Einav, Finkelstein, and Shripf (2010) who find preference for bequest has high positive correlation with high mortality. Nevertheless, the results are reported here to show sensitivity with respect to this parametrization. This leads to increase (decrease) in demand for annuity by higher (lower) mortality types in the model as we increase the value of parameter  $\varpi$ —everything else being equal.

Table 10 show calibrated values for  $\xi$  for different values of intercept parameter  $\varpi$ . The fraction who purchase annuity is lower for higher values of  $\varpi$ . Table 10 shows the results of welfare calculations. Welfare gains are higher for positive values of  $\varpi$ . However, in general results of the paper are still valid. Welfare gains from social security annuitization are much smaller when annuity markets exist and respond to policy (relative to autarky case). Also, social security has a crowding out effect that leads to an increase in annuity prices. This, in turn lowers welfare gains to lower mortality types and limits the potential gain from the policy.

TABLE 10: Calibrated Bequest Parameter for Values of  $\varpi$

Calibrated weight on bequest				
(Chosen to match average annuitized wealth of 10% at age 65–70)				
	$\varpi = 0^*$	$\varpi = 1$	$\varpi = 5$	$\varpi = 10$
$\xi$	0.9	2.17	21.35	76.44
Fraction with annuity (not targeted)	0.53	0.43	0.36	0.32

*Note* For each value of risk aversion parameter  $\xi$  is chosen to such that average fraction of annuitized wealth at retirement is 10%. All other parameters are the same as benchmark (Table 4 in the paper.)

\* Benchmark calibration.

TABLE 11: Welfare Calculations for Non-homothetic Preferences

	$\varpi = 0^*$	$\varpi = 1$	$\varpi = 5$	$\varpi = 10$
	$\xi = 0.9$	$\xi = 2.17$	$\xi = 21.35$	$\xi = 76.44$
	Ex ante welfare gains from annuitization in the current US system (%)			
Autarky	2.68	3.27	2.56	1.94
Annuity Market with Full Info.	0.01	0.47	0.32	0.25
Annuity Market with Private Info.	0.07	0.53	0.39	0.34
Annuity Market with Private Info. (with annuity prices fixed)	0.42	0.95	0.77	0.56
	Welfare losses due to private information (%)			
Without Social Security	0.38	0.43	0.44	0.44
With Social Security	0.32	0.37	0.37	0.35
	Welfare gains from having access to annuity market (%)			
Without Social Security	2.79	2.87	2.28	1.69
With Social Security	0.17	0.14	0.11	0.10
	Ex ante welfare gains from implementing the first best (%)			
Autarky	4.32	4.00	3.16	2.53
Annuity Market with Full Info.	1.11	0.65	0.42	0.37
Annuity Market with Private Info.	1.49	1.09	0.86	0.82

\* Benchmark calibration.

## H Annuitization in HRS

This section contain Table 1 and 2 of Section 4.2 of the paper by marital status and age.

TABLE 12: Average Fraction of Retirement Wealth that is Annuitized

	Employer Sponsored Pensions (%)	Private Annuities (%)	Social Security (%)
All	9.5	0.5	34.5
60-64	11.8	0.3	41.2
65-69	11.0	0.3	37.0
70-74	10.4	0.6	37.5
75-79	8.8	0.7	33.2
80+	6.0	0.7	26.4
Single			
60-64	10.8	0	49.8
65-69	9.2	0.2	37
70-74	9.5	0.5	49
75-79	11.5	0.7	39.3
80+	6.6	0.8	31.6
Married			
60-64	12.1	0.4	39.2
65-69	11.4	0.3	37
70-74	10.6	0.6	35
75-79	8.2	0.6	31.8
80+	5.6	0.6	23.3

*Source:* Author's calculations using wave 7 of RAND HRS data, version M.

*Notes:* All calculations are for male respondents who are older than 60 and collecting social security retirement benefit in wave 7.

TABLE 13: Fraction of Population with Private Annuity or Defined Benefit Pension

	In their own name (%)			In own or spouse's name (%)		
	Emp.-spons. Pension	Private Annuity	Pension or Annuity	Emp.-spons. Pension	Private Annuity	Pension or Annuity
All	45.6	4.8	48.0	51.5	6.2	53.9
60-64	40.0	1.8	40.5	51.5	6.2	53.9
65-69	41.8	2.5	42.9	47.5	3.3	48.7
70-74	46.0	4.5	48.5	52.6	6.1	54.9
75-79	49.7	5.6	52.7	56.7	7.0	59.7
80+	48.9	8.8	52.9	54.4	10.8	58.7
<hr/>						
Single						
60-64	37	0	37			
65-69	36.1	1.7	37.4			
70-74	34.5	4.9	37.4			
75-79	50.6	9.2	54.9			
80+	48.6	10	53.6			
<hr/>						
Married						
60-64	40.7	2.3	41.3	45.3	2.8	45.9
65-69	42.9	2.6	44	49.9	3.7	51.1
70-74	48.5	4.4	50.8	56.5	6.4	58.6
75-79	49.6	4.8	52.2	58.1	6.5	60.8
80+	49	8.1	52.5	57.8	11.4	61.7

*Source:* Author's calculations using wave 7 of RAND HRS data, version L.

*Notes:* All calculations are for male respondents who are older than 60 and collecting social security retirement benefit in wave 7.

## References

- Ameriks, John, Andrew Caplin, Steven Laufer, and Stijn Van Nieuwerburgh. 2011. “The joy of giving or assisted living? Using strategic surveys to separate public care aversion from bequest motives.” *The journal of finance* 66 (2):519–561.
- Bisin, Alberto and Piero Gottardi. 2006. “Efficient Competitive Equilibria with Adverse Selection.” *Journal of Political Economy* 114:485–516.
- Brown, Jeffrey R, T. Davidoff, and Peter Diamond. 2005. “Annuities and Individual Welfare.” *American Economic Review* 95:1573–90.
- Cannon, Edmund and Ian Tonks. 2008. *Annuity markets*. Oxford University Press.
- De Nardi, Mariacristina. 2004. “Wealth inequality and intergenerational links.” *The Review of Economic Studies* 71 (3):743–768.
- Dionne, G., N. Doherty, , and N. Fombaron. 2000. “Adverse Selection in Insurance Markets.” *Handbook of Insurance* .
- Einav, Liran, Amy Finkelstein, and Paul Shrimp. 2010. “Optimal Mandates and The Welfare Cost of Asymmetric Information: Evidence from The U.K. Annuity Market.” *Econometrica* 78(3):1031–1092.
- Finkelstein, Amy and James Poterba. 2004. “Adverse Selection in Insurance Markets: Policyholder Evidence from the U.K. Annuity Market.” *Journal of Political Economy* 112 (1):183–208.
- Lockwood, Lee M. 2012. “Bequest Motives and the Annuity Puzzle.” *Review of Economic Dynamics* 15 (2):226–243.
- Milgrom, P. and J. Roberts. 1994. “Comparing equilibria.” *The American Economic Review* :441–459.
- Netzer, Nick and Florian Scheuer. forthcoming. “A game theoretic foundation of competitive equilibria with adverse selection.” *International Economic Review* .
- Pashchenko, S. 2013. “Accounting for Non-Annuity.” *Journal of Public Economics* 98:53–67.
- Rothschild, Michael and Joseph E Stiglitz. 1976. “Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information.” *Quarterly Journal of Economics* 90 (4):630–49.