Tariff Policy with an Exorbitant Privilege*

David Andolfatto
University of Miami

Roozbeh Hosseini
University of Georgia

March 17, 2026

Abstract

This paper studies tariff policy in a reserve-currency economy. We develop a two-country dynamic general equilibrium model in which one country supplies the internationally demanded transaction asset. In equilibrium, foreign demand for this asset is financed by net shipments of tradables to the issuer, generating a persistent trade deficit and a long-run contraction of the issuer's tradables sector. Motivated by recent policy proposals—including those associated with Miran Miran (2024) and the so-called "Mar-a-Lago Accord"—that advocate broad import tariffs (sometimes combined with offsetting fiscal rebates and international negotiations) as a route to reduce the U.S. trade deficit and "reindustrialize" the economy, we use the model to evaluate the effects of unilateral import tariffs. In the benchmark environment, a tariff does not change long-run tradables employment or output, and it does not change the long-run quantity of tradables imported; instead it alters relative prices and therefore the measured trade deficit (a valuation effect), while raising fiscal revenue. The long-run burden of the tariff is shared: part is borne abroad through lower net-of-tariff receipts and part at home through higher consumer prices. The main implication is that, in a reserve-currency environment, tariffs are primarily a revenue-shifting device and a way to change relative prices, not a reliable instrument for sustained trade rebalancing or tradables-sector expansion.

JEL Classification: F13, F32, F41, E42.

Keywords: reserve currency; exorbitant privilege; tariffs; trade deficit.

*We thank participants at the Third Jackson Hole International Finance and Macroeconomics Conference for useful comments and suggestions. Remaining errors are ours.

1 Introduction

The United States runs a persistent trade deficit. Policymakers routinely describe this as a problem to be solved and reach for tariffs as the instrument. Before asking whether tariffs can solve it, we should ask what the deficit actually is. Our answer is that it is the equilibrium counterpart of foreign demand for U.S.-issued transaction assets—not a distortion, not a sign of unfair competition, but the price the rest of the world pays to hold dollars. If that is right, the question of whether tariffs can eliminate the deficit becomes a question about whether tariffs can eliminate the demand for dollars. The answer, we will argue, is no.

We call this the "exorbitant privilege" setting. A country with exorbitant privilege can issue claims—safe, liquid, dollar-denominated—that the rest of the world is willing to hold at low yields. The counterpart of this privilege is a real transfer: to accumulate those claims, foreigners must ship goods in exchange. This transfer is not a bookkeeping curiosity. It is what determines relative prices between tradable and nontradable goods at home, and through those prices, the long-run allocation of labor across sectors. A tariff operates on those same prices. The question is whether it can undo what asset demand has done.

The current policy debate takes a different view. Proposals associated with Miran Miran (2024) and the so-called "Mar-a-Lago Accord" treat the deficit as a price distortion and tariffs as the corrective instrument—broad import levies, sometimes paired with domestic rebates of tariff revenue and, in some versions, international negotiations over exchange rates or reserve-asset arrangements—designed to divert spending toward domestic producers and shrink the import bill. Part of the appeal is distributional: some of the tariff, the argument goes, will be "paid by foreigners" through compressed export prices. This logic is correct in partial equilibrium. The question is whether it survives in a general equilibrium where the trade deficit is not a price distortion but the market-clearing outcome of foreign asset demand.

We address this question in a stylized two-country model. The two economies are otherwise identical; what distinguishes them is that the home country issues the only internationally accepted transaction asset. Households in both countries consume a tradable good and a nontradable service. Production is Ricardo–Viner: each sector has a sector-specific factor and a mobile factor that can move between them. Foreign demand for the home transaction asset is financed, in equilibrium, by net shipments of tradables to the home country.

The equilibrium has a natural representation in terms of labor-market diagrams. A labor supply schedule captures the condition under which the mobile factor is indifferent between sectors; a labor demand schedule captures how expenditure patterns and prices pin down demand for tradables production. In the reserve-currency equilibrium, foreign asset demand shifts these schedules so that the home country runs persistent net imports and allocates less mobile labor to tradables in the long run.

The main findings follow directly from this structure. The foreign economy's decision about how many tradables to export is governed entirely by its demand for the home transaction asset—not by the tariff wedge. This pins down both the quantity of foreign production allocated to home imports and the foreign labor allocation, independently of any tariff the home country might impose. At home, the tariff raises the consumer price of imported goods, but when tariff revenue is rebated lump-sum to consumers, their spending power rises by exactly the same amount. The home labor market sees no change in the relative return to producing tradables; the home labor allocation is unchanged. The long-run import quantity, home sectoral employment, and foreign sectoral employment are all invariant to the tariff.

What does change is relative prices. A higher tariff drives a wedge between the home consumer price and the foreign producer price. Since trade statistics value imports at world prices while home GDP is valued at domestic prices, this wedge mechanically reduces the measured trade-deficit-to-GDP ratio—not because fewer goods are imported, but because imported goods appear cheaper relative to home output. The tariff is, in this sense, a valuation device rather than a rebalancing device.

The incidence is shared. Foreign exporters receive lower net-of-tariff prices; home consumers pay more for imported goods. What looks like a tax on foreigners is a tax shared between domestic consumers and foreign producers, mediated by the general equilibrium price response. None of this implies that tariffs are irrelevant—they raise revenue and shift the international distribution of income. What they cannot do, in a reserve-currency setting, is eliminate the trade deficit or expand the tradables sector, because those outcomes are governed by asset demand, not by the price of imports.

1.1 Related literature

Three bodies of work bear directly on what we are doing here.

The first concerns the macroeconomic consequences of the dollar's special role. Gourinchas and Rey Gourinchas and Rey (2005, 2013) document that the United States functions as a world banker and that external adjustment occurs through valuation effects as well as trade flows. Caballero, Farhi, and Gourinchas Caballero et al. (2017, 2016) rationalize persistent imbalances as the equilibrium outcome of global safe-asset scarcity; Krishnamurthy and Vissing-Jorgensen Krishnamurthy and Vissing-Jorgensen (2012) document the convenience yield that sustains demand for U.S. Treasuries empirically. The closest theoretical predecessor to our paper is Farhi and Maggiori Farhi and Maggiori (2018), who build a formal model of the international monetary system in which a single reserve-currency issuer supplies safe assets to the rest of the world. We share their core asymmetry but work in a stripped-down two-sector environment that lets us trace reserve-currency demand through to sectoral labor reallocation—a connection their framework does not emphasize. An influential alternative interpretation of the U.S. external deficit is Bernanke's Bernanke (2005) "global saving glut" hypothesis, which attributes persistent imbalances to high desired saving in emerging markets rather than to a demand for safe assets per se. The two narratives are not mutually exclusive—both imply net resource flows to the United States—but our model captures the transaction-asset demand channel specifically, following the safe-asset literature. On global imbalances more broadly, see Obstfeld and Rogoff Obstfeld and Rogoff (2005). The dominant-currency paradigm of Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller Gopinath et al. (2020) documents a related but distinct dimension of dollar dominance—trade invoicing—which amplifies the mechanisms we study through pricing complementarities.

The second body of work is the classical theory of trade policy. The Lerner symmetry theorem Lerner (1936) establishes that broad-based import tariffs and export taxes are equivalent, limiting tariffs' ability to improve the trade balance without accompanying changes in saving or investment. The optimal-tariff tradition Metzler (1949); Bagwell and Staiger (1999, 2002) shows that a large country can improve its terms of trade through a tariff. Our model uses a specific-factors (Ricardo–Viner) production structure Jones (1971), which makes the sectoral reallocation implications of tariffs transparent. We show how the classical results interact with the asset-demand mechanism: terms-of-trade movements occur, but they operate through the valuation channel rather than through quantity rebalancing.

The third strand quantifies tariff pass-through and incidence. The 2018–2019 trade war has generated a detailed record. Amiti, Redding, and Weinstein Amiti et al. (2019) and Fajgelbaum, Goldberg, Kennedy, and Khandelwal Fajgelbaum et al. (2020) document substantial pass-through of tariffs into domestic prices, with burden shared between domestic consumers and foreign exporters. Our model generates exactly this shared-incidence pattern and connects it to the reserve-currency mechanism.

A fourth strand motivates the sectoral dimension of our analysis. Autor, Dorn, and Hanson Autor et al. (2013) document the local labor market effects of import competition in the United States, showing that trade exposure causes persistent reallocation away from tradables—the deindustrialization pattern that our model produces as an equilibrium outcome of reserve-currency demand. Their evidence gives empirical content to the sectoral reallocation we characterize theoretically.

Most directly related are Caliendo, Kortum, and Parro Caliendo et al. (2025), who emphasize valuation channels in interpreting tariff-induced changes in measured trade deficits, and Itskhoki and Mukhin Itskhoki and Mukhin (2025), who characterize the optimal "macro tariff" in an environment with large gross external positions. We share their emphasis on valuation and the interaction between trade policy and the macro balance sheet. Our contribution is to isolate the reserve-currency asymmetry as the key structural feature: in our model the real volume of net imports is determined by asset demand, not by prices, and this is what makes the tariff neutral in the long run.

Road map. Section 2 develops the model, beginning with a closed economy and then introducing the reserve-currency open economy; it also characterizes transition dynamics. Section 3 analyzes tariff policy in the benchmark environment, establishing the long-run neutrality result (Proposition 2), the valuation channel (Section 3.2), and the incidence of the tariff burden (Section 3.3). Section 4 presents three extensions: imperfect substitutability between home and foreign tradables (Section 4.1), domestic industrial policy (Section 4.2), and alternative fiscal dispositions of tariff revenue (Section 4.3). Section 5 concludes. Appendix A collects all proofs. Appendix C develops the symmetric Armington extension.

2 Model

2.1 Closed Economy

We begin with a closed economy. Agents live for two periods. The young work; the old consume. There are two production sectors—goods and services—and three types of workers. Type 1 is specific to goods, type 2 is specific to services, type 3 is mobile and can work in either. There is one unit of each type.

Let c_t denote services and let k_t denote goods. Let $0 \leq n_t \leq 1$ denote the measure of type 3 (mobile) labor employed in the goods sector. Goods are produced using input from the type 1 worker (fixed) and input from the type 3 worker according to the production function $k_t = n_t^\alpha$. Since input of type 1 is fixed, we suppress it as an argument and treat it as a factor in fixed supply. Services are produced using input from type 2 and type 3 workers according to $c_t = G\left(1 - n_t\right)$. Let p_t^c and p_t^k denote the money price of services and goods, respectively, and let w_t^3 be the wage of the mobile worker. Indifference of the mobile worker across sectors requires

$$w_t^3 = p_t^c G'\left(1 - n_t\right) = p_t^k F'\left(n_t\right)$$

For simplicity, assume $G = F$ and $F\left(n\right) = n^\alpha$, $0 < \alpha < 1$, so that $F'\left(n\right) = \alpha F\left(n\right)/n$ and

$F(n) - nF'(n) = (1-\alpha) F(n)$.[1] The wages paid to each type are then

$$w_t^1 = p_t^k (1-\alpha) n_t^\alpha \tag{1}$$

$$w_t^2 = p_t^c (1-\alpha)(1-n_t)^\alpha \tag{2}$$

$$w_t^3 = p_t^k \alpha n_t^{\alpha-1} = p_t^c \alpha (1-n_t)^{\alpha-1} \tag{3}$$

Individuals have identical preferences over future goods and services,

$$U_t = u(c_{t+1}, k_{t+1}).$$

Young households do not value current consumption, so they save all their income. Type 1 and 2 households have a trivial labor supply choice. Type 3 households are indifferent in equilibrium across sectors and earn the same wage w_t^3 in either one.[2]

Goods and services are non-storable. The single asset is nominal government debt (fiat money). Let M_t denote the money supply at date t, where $M_t = \mu M_{t-1}$ with M_0 in the hands of the initial old. New money $\tau_t = (1 - 1/\mu) M_t$ is injected as a lump-sum transfer to the old and divided equally among types. Let m_t^i be young household i's money holdings.

Definition 1. *A closed economy competitive equilibrium is a sequence of consumption allocations and money holdings* $\{c_t^i, k_t^i, m_t^i\}_{t=0,i\in\{1,2,3\}}^\infty$*, a fraction of type 3 households in the goods sector* $\{n_t\}_{t=0}^\infty$*, prices* $\{p_t^c, p_t^k\}_{t=0}^\infty$*, wages* $\{w_t^i\}_{t=0,i\in\{1,2,3\}}^\infty$*, and a government policy* $\{M_t, \tau_t\}_{t=0}^\infty$*, such that*

1. *Given prices, wages and government policy, allocation* $\{c_t^i, k_t^i\}_{t=0}^\infty$ *solves the problem of individual worker* i*'s decision problem* $\max u(c_{t+1}^i, k_{t+1}^i)$ *subject to:*

$$m_t^i \leq w_t^i \text{ and } p_{t+1}^c c_{t+1}^i + p_{t+1}^k k_{t+1}^i \leq m_t^i + \frac{\tau_{t+1}}{3}$$

2. *The wages are determined by equations (1), (2) and (3);*
3. *The government budget constraint holds:* $\tau_t = M_t - M_{t-1}$*;*
4. *Markets clear:* $\sum_{i=1,2,3} c_t^i = (1-n_t)^\alpha$; $\sum_{i=1,2,3} k_t^i = n_t^\alpha$; $\sum_{i=1,2,3} m_t^i = M_t$

In any equilibrium, the following accounting identity must hold:

$$p_t^k n_t^\alpha + p_t^c (1-n_t)^\alpha = M_t = w_t^1 + w_t^2 + w_t^3,$$

[1] Different types of labor are not substitutes for one another.

[2] Technically, the young value their time a little bit, to avoid having them supply labor at zero wage rates.

where the left side is nominal GDP and the right side is national income. With Cobb-Douglas preferences $u(c,k) = c^{1-\phi}k^{\phi}$, household optimization implies:

$$p_t^k k_t^i = \phi \left[w_{t-1}^i + \frac{\tau_t}{3} \right] \text{ and } p_t^c c_t^i = (1-\phi) \left[w_{t-1}^i + \frac{\tau_t}{3} \right]$$

Since old households spend their entire nominal wealth M_t on goods and services:

$$p_t^k n_t^{\alpha} = \phi M_t \text{ and } p_t^c (1-n_t)^{\alpha} = (1-\phi) M_t$$

which implies:

$$\frac{p_t^k}{p_t^c} = \frac{\phi}{1-\phi} \frac{F(1-n_t)}{n_t^{\alpha}}$$

From (3):

$$\frac{p_t^k}{p_t^c} = \frac{n_t}{1-n_t} \frac{F(1-n_t)}{n_t^{\alpha}}$$

Combining the previous two equations yields:

$$n_t = \phi$$

so that,

$$\frac{p_t^k}{p_t^c} = \frac{\phi}{1-\phi} \frac{F(1-\phi)}{\phi} \equiv \xi(\phi)$$

2.1.1 A labor supply–labor demand representation

The closed-economy equilibrium allocation of the mobile factor n_t admits a simple graphical characterization that will be useful later.

Labor supply (sectoral choice). The mobile factor (type 3) is indifferent across sectors in equilibrium. Using $n^{\alpha} = n^{\alpha}$, the no-arbitrage condition $w_t^3 = p_t^k F'(n_t) = p_t^c F'(1-n_t)$ implies

$$\frac{p_t^k}{p_t^c} = \left(\frac{1-n_t}{n_t} \right)^{\alpha-1}. \tag{4}$$

We interpret (4) as a *labor supply schedule* to the goods (tradables) sector: a higher relative price p_t^k/p_t^c raises the relative return to working in the goods sector and increases n_t.

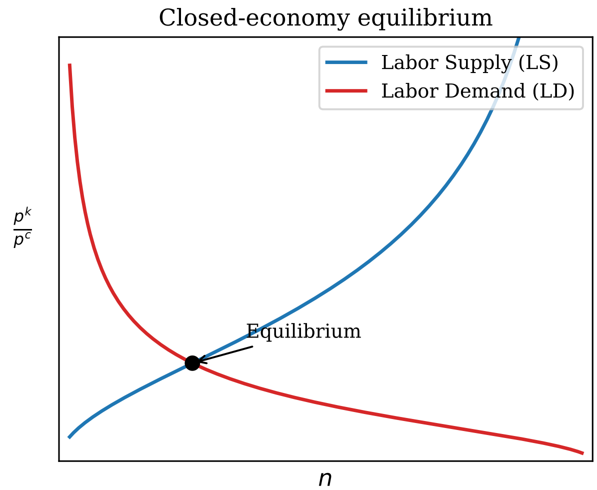


Figure 1: Closed-economy equilibrium as the intersection of a labor supply schedule (4) (blue curve; sectoral choice of the mobile factor) and a labor demand schedule (5) (red curve; expenditure shares). The horizontal axis is the mobile labor share in the goods sector, n, and the vertical axis is the relative price p^k/p^c.

Labor demand (expenditure shares). Using old-age expenditure shares, $p_t^k n_t^\alpha = \phi M_t$ and $p_t^c F(1-n_t) = (1-\phi)M_t$. Combining these conditions yields

$$\frac{p_t^k}{p_t^c}\frac{n_t^\alpha}{(1-n_t)^\alpha} = \frac{\phi}{1-\phi}. \tag{5}$$

We interpret (5) as a *labor demand schedule* for the goods sector: for a given n_t, the relative price must adjust so that households devote fraction ϕ of nominal spending to goods.

Equilibrium. The intersection of (4) and (5) delivers $n_t = \phi$.

From nominal GDP we can derive prices:

$$p_t^c = \frac{M_t}{\xi(\phi)\,\phi^\alpha + (1-\phi)^\alpha}$$
$$p_t^k = \frac{\xi(\phi)\,M_t}{\xi(\phi)\,\phi^\alpha + (1-\phi)^\alpha}$$

Sector prices grow at rate μ. The aggregate price level is

$$p_t = \phi p_t^k + (1-\phi)\,p_t^c$$

and the equilibrium inflation rate is μ.

Real allocations in this closed economy are invariant to inflation. There is no savings decision and no portfolio allocation problem; inflation scales all nominal wages and prices uniformly. What inflation does is finance the lump-sum transfers to the old—that is its only role here.

2.2 Open Economy

Now add a second country, identical to the first in every respect except one: it has no financial assets. No money, no private financial markets. In autarky this foreign economy cannot function—there is no medium of exchange and hence no production. But open international trade creates an opportunity. The foreign economy has labor and technology; the home economy has the only transaction asset. The stage is set for an exchange.

What do we expect when trade opens? Goods are tradable; services are not. Home and foreign tradables are perfect substitutes (we relax this in Section 4.1), so the real exchange rate between them is fixed at par. There is a single currency, so there is no nominal exchange rate to worry about.

Let foreign allocations be denoted with an asterisk ($*$). Let p_t^c and p_t^{c*} denote the price of services at home and abroad. Goods are tradable with a common price p_t^k.

Definition 2. *An open economy competitive equilibrium is a sequence of consumption allocations and money holdings in domestic and foreign countries* $\{(c_t^i, k_t^i, m_t^i), (c_t^{i*}, k_t^{i*}, m_t^{i*})\}_{t=0, i\in\{1,2,3\}}^{\infty}$, *the fraction of type 3 households in the goods sector* $\{n_t, n_t^*\}_{t=0}^{\infty}$, *prices* $\{p_t^c, p_t^{c*}, p_t^k\}_{t=0}^{\infty}$, *wages* $\{w_t^i, w_t^{i*}\}_{t=0, i\in\{1,2,3\}}^{\infty}$, *and government policy* $\{M_t, \tau_t\}_{t=0}^{\infty}$, *such that*

1. *Given prices, wages and government policy, allocation* $\{c_t^i, k_t^i, m_t^i\}_{t=0}^{\infty}$ *solves the problem of domestic household* $i = 1, 2, 3$*:* $\max u\left(c_{t+1}^i, k_{t+1}^i\right)$ *subject to:*

$$m_t^i \leq w_t^i \text{ and } p_{t+1}^c c_{t+1}^i + p_{t+1}^k k_{t+1}^i \leq m_t^i + \frac{\tau_{t+1}}{3}$$

2. *Given prices, wages and government policy, allocation* $\{c_t^{i*}, k_t^{i*}, m_t^{i*}\}_{t=0}^{\infty}$ *solves the problem of foreign household* i *(the difference is that they do not receive the transfer):* $\max u\left(c_{t+1}^{i*}, k_{t+1}^{i*}\right)$, *subject to:*

$$m_t^{i*} \leq w_t^{i*} \text{ and } p_{t+1}^{c*} c_{t+1}^{i*} + p_{t+1}^k k_{t+1}^{i*} \leq m_t^{i*}$$

3. *Domestic wages are determined according to equations (1), (2) and (3) (and similarly for foreign wages).*

4. *The government budget constraint holds:* $\tau_t = M_t - M_{t-1}$;

5. *Markets clear*

$$\sum_{i=1,2,3} c_t^i = (1-n_t)^\alpha$$
$$\sum_{i=1,2,3} c_t^{i*} = (1-n_t^*)^\alpha$$
$$\sum_{i=1,2,3} k_t^i + \sum_{i=1,2,3} k_t^{i*} = n_t^\alpha + n_t^{*\alpha}$$
$$\sum_{i=1,2,3} m_t^i + \sum_{i=1,2,3} m_t^{i*} = M_t$$

The equilibrium can be reduced to a system of three equations, as the following lemma states.

Lemma 1. *In the open economy, competitive equilibrium is characterized by the following system of equations.*

$$(1-n_t^*)\, n_t^{*\alpha-1} = (1-\phi)\,\frac{p_{t-1}^k}{p_t^k} n_{t-1}^{*\alpha-1} \tag{6}$$
$$(1-n_t)\, n_t^{\alpha-1} = (1-\phi)\,\frac{p_{t-1}^k}{p_t^k}\left[\mu n_{t-1}^{\alpha-1} + (\mu-1)\, n_{t-1}^{*\alpha-1}\right] \tag{7}$$
$$n_t^\alpha + n_t^{*\alpha} = \phi\frac{p_{t-1}^k}{p_t^k}\mu\left[n_{t-1}^{\alpha-1} + n_{t-1}^{*\alpha-1}\right] \tag{8}$$
$$\tag{9}$$

and

$$p_0^c\,(1-n_0)^\alpha = (1-\phi)\, M_0 \tag{10}$$
$$p_0^k\,(n_0^{*\alpha} + n_0^\alpha) = \phi M_0 \tag{11}$$

$$\frac{p_0^k}{p_0^c} = \frac{n_0}{1-n_0}\frac{(1-n_0)^\alpha}{n_0^\alpha} \tag{12}$$

with M_0 *given and* $n_0^* = 1$

Proof. See Appendix A.1. □

Interpretation of the equilibrium conditions. Equation (6) is the *foreign service-market clearing condition*: the value of services produced abroad equals the fraction $(1-\phi)$ of foreigners' nominal income, with the ratio p_{t-1}^k/p_t^k capturing goods-price inflation. This equation pins down n_t^* given the history (n_{t-1}^*) and the price path.

Equation (7) is the *home service-market clearing condition*. Home residents spend fraction $(1-\phi)$ of their total nominal income on services. The term μ multiplying home wages reflects money injected each period (seigniorage), while $(\mu - 1)$ times foreign wages captures seigniorage collected from foreign money holdings. This extra income is the quantitative heart of the exorbitant privilege.

Equation (8) is *global goods-market clearing*: world tradables production equals world demand, where demand is fraction ϕ of world nominal income (scaled by μ to account for new money).

Together, the three equations determine the sequence $(n_t, n_t^*, p_{t-1}^k/p_t^k)$, tracing out how the global labor allocation and price path co-evolve.

The opening period is stark. Home's initial old hold all the money; they can buy goods and services, and production proceeds normally in both sectors. The foreign old have nothing—no money, no claims—and can buy nothing. Since foreign services cannot be traded and no one in the foreign economy can pay for them, no services are produced there in period 0. Every mobile worker in the foreign economy goes into goods production; type 2 workers—specific to services—are idle. The entire foreign goods output is shipped to the home economy in exchange for money. This is the transaction that sets everything in motion, and it is why the initial condition is $n_0^* = 1$. Combining the initial conditions gives

$$\frac{n_0}{1-n_0}\frac{1+n_0^\alpha}{n_0^\alpha} = \frac{\phi}{1-\phi}$$

Importantly, n_0 does not depend on μ. And $n_0 < \phi$: the immediate impact of opening trade is a contraction of home goods production, as the flood of cheap foreign goods lowers the relative return to producing tradables at home.

Given the initial labor allocation, the initial prices are:

$$p_0^c = \frac{(1-\phi)\,M_0}{(1-n_0)^\alpha}$$
$$p_0^k = \frac{\phi M_0}{1+n_0^\alpha}$$

Let m_t^d and m_t^{d*} be the money that young households hold at the end of period t, with $m_t^d + m_t^{d*} = M_t = \mu^t M_0$. In the initial period:

$$m_0^d = \frac{\phi M_0 n_0^\alpha}{1+n_0^\alpha} + (1-\phi)\,M_0$$

and

$$m_0^{d*} = \frac{\phi M_0}{1+n_0^\alpha}$$

Let GDP_t and GDP_t^* denote nominal GDP in the home and foreign economies, and EXP_t and EXP_t^* aggregate nominal expenditure (total spending by the old, inclusive of seigniorage transfers):

$$\mathrm{GDP}_t = p_t^k n_t^\alpha + p_t^c \left(1 - n_t\right)^\alpha$$
$$\mathrm{GDP}_t^* = p_t^k n_t^{*\alpha} + p_t^{c*} \left(1 - n_t^*\right)^\alpha$$

$$\mathrm{EXP}_t = m_{t-1}^d + (\mu - 1)\left(m_{t-1}^d + m_{t-1}^{d*}\right)$$
$$\mathrm{EXP}_t^* = m_{t-1}^{d*}$$

2.2.1 Steady State

Define a steady state as a situation where $n_t = n, n_t^* = n^*$ and $p_t^c/p_{t-1}^c = p_t^{c*}/p_{t-1}^{c*} = p_t^k/p_{t-1}^k = \mu$.

Three properties of the steady state are worth stating before we prove them. First, the reserve-currency asymmetry reallocates mobile labor in opposite directions in the two countries: the foreign economy tilts toward goods, the home economy toward services. Second, this reallocation generates wage inequality between sector-specific workers—inequality rises at home, falls abroad. Third, the home economy runs a persistent trade deficit. These are not separate phenomena; they are all consequences of the same underlying force: foreign demand for the home transaction asset.

Let us define wage inequality as the ratio of the wage of a service producer to the wage of the goods producer.

Proposition 1. *Suppose $\mu \geq 1$. In steady state, the following statements are true:*

1- $n^* \geq \phi \geq n$, with equality if and only if $\mu = 1$;

2- $\frac{w_t^2}{w_t^1} \geq \frac{1-\phi}{\phi} \geq \frac{w_t^{2*}}{w_t^{1*}}$, with equality if and only if $\mu = 1$;

3- $\dfrac{\mathrm{EXP}_t}{\mathrm{GDP}_t} \geq 1 \geq \dfrac{\mathrm{EXP}_t^*}{\mathrm{GDP}_t^*}$, with equality if and only if $\mu = 1$.

Proof. See Appendix A.2. □

As long as money grows, the foreign economy produces more tradables than in autarky and ships the excess home in exchange for money. The home economy absorbs those shipments and tilts its mobile labor toward services. The relative wage of goods workers falls at home; it rises abroad. Trade never balances as long as money is growing: the home economy always consumes more than it produces, paying with claims on future seigniorage. Even if money growth stops, the accumulated foreign money balances remain outstanding—the debt is never paid, it is simply held.

Why does the foreign economy tilt toward goods? Because goods are the only tradable object it can deliver in exchange for money. Why does the home economy tilt toward services? Because access to cheap imported goods lowers the relative profitability—and the relative wage—of producing tradables at home, drawing mobile labor into the nontradables sector. The two effects are the same phenomenon seen from opposite sides.

2.2.2 Steady-state labor supply and labor demand

The steady-state reallocation $n < \phi < n^*$ has a transparent interpretation in terms of a labor-supply/labor-demand diagram, analogous to Figure 1.

Labor supply (sectoral choice). In both countries, the mobile factor must be indifferent across sectors. With $n^\alpha = n^\alpha$, this delivers the same upward-sloping "labor supply" schedule as in the closed economy:

$$\frac{p^k}{p^c} = \left(\frac{1-n}{n}\right)^{\alpha-1}, \qquad \frac{p^k}{p^{c*}} = \left(\frac{1-n^*}{n^*}\right)^{\alpha-1}. \tag{13}$$

Labor demand (asset-demand wedge). What differs across countries is the schedule linking relative prices to expenditure shares once the reserve-currency asymmetry is taken into account.

In the foreign economy, steady-state goods-market clearing and the fact that prices grow at rate μ imply the "labor demand" condition

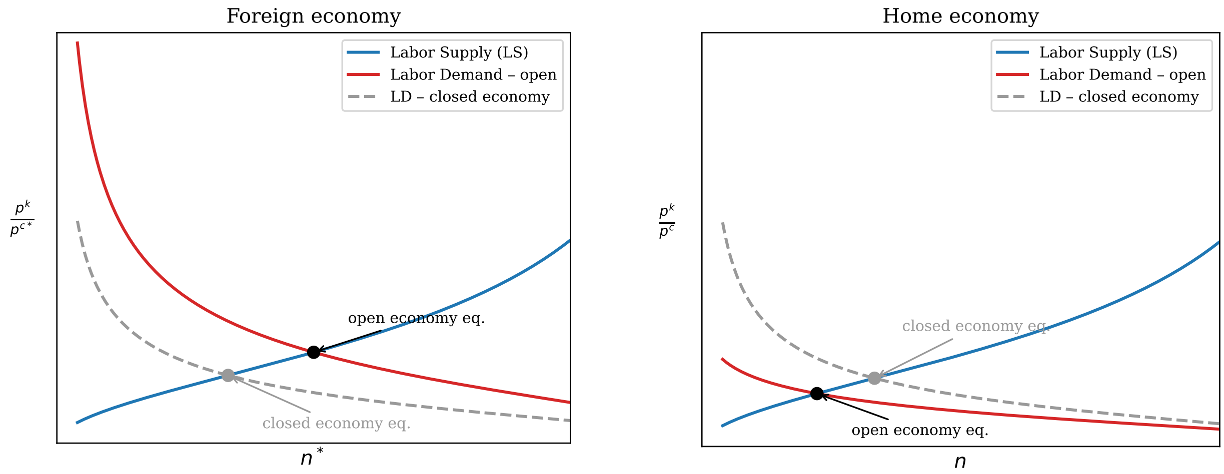
$$\frac{(1-n^*)^\alpha}{n^{*\,\alpha-1}} = \frac{1-\phi}{\mu}\,\frac{p^k}{p^{c*}}. \tag{14}$$

Because the foreign economy must acquire the transaction asset by exporting tradables, the shadow value of producing tradables is high, shifting the foreign labor-demand curve up relative to autarky.

In the home economy, the analogous steady-state condition becomes

$$\frac{p^k}{p^c} = \frac{\phi}{1-\phi}\,\frac{(1-n)^\alpha}{\left(1-\frac{1}{\mu}\right) n^{*\,\alpha-1} + n^\alpha}. \tag{15}$$

The term $\left(1-\frac{1}{\mu}\right) n^{*\,\alpha-1}$ captures the additional nominal spending power accruing to home residents from money growth on a balance sheet partly held abroad—seigniorage from foreign asset demand. This shifts the home labor-demand curve down, reducing the relative price of tradables and drawing mobile labor into nontradables.



(a) Foreign economy.

(b) Home economy.

Figure 2: Open-economy steady state as an intersection of labor supply (blue curve; (13)) and labor demand (red curve; (14)–(15)). The dashed gray curve is the closed-economy labor demand (5), and the gray dot is the closed-economy equilibrium. The reserve-currency asymmetry shifts the foreign labor-demand schedule up (raising n^* above the closed-economy benchmark) and shifts the home labor-demand schedule down (lowering n below the closed-economy benchmark).

2.2.3 Transition

The steady-state results describe long-run allocations. The transition dynamics, which we cannot characterize analytically, are illustrated in Figure 3 for the parameter values $\phi = 0.2$, $\alpha = 0.5$, $\mu = 1.05$.

The opening period is the starkest. The foreign economy, eager to acquire money the moment trade opens, exports its entire goods output to the home economy. The trade deficit jumps immediately to its maximum. If there is no money growth, the foreign economy has all the money it needs after one period and stops exporting; the deficit falls to zero and the economies thereafter operate as two closed economies, the foreign country holding its money balances forever.

With money growth, the story is different. The foreign economy must continuously export to replenish the real value of money holdings eroded by inflation. The economies transition gradually to the steady state described above. The trade deficit declines from its initial peak but remains positive throughout. The wage premium for service workers in the home economy follows the same pattern: highest on impact, declining toward its steady-state value.

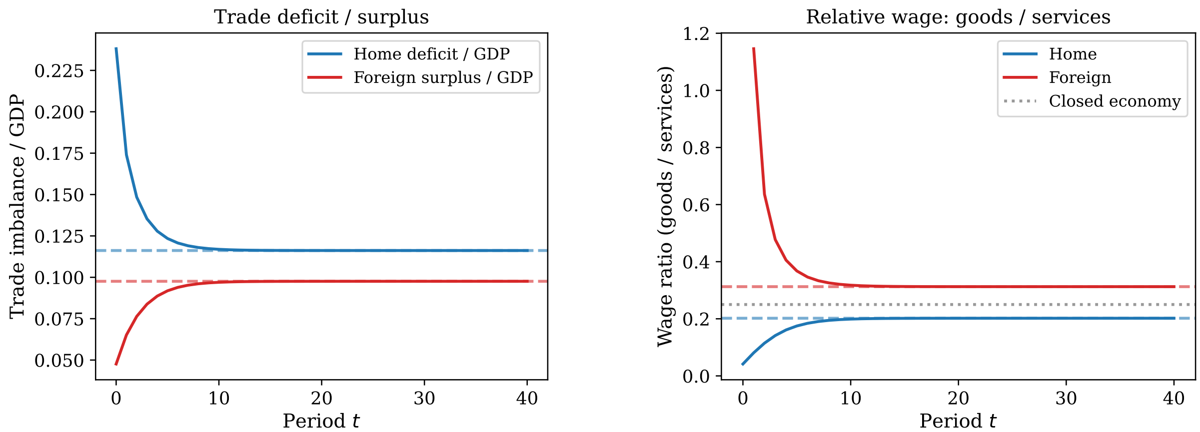


Figure 3: Transition dynamics following the opening of trade ($\phi = 0.2$, $\alpha = 0.5$, $\mu = 1.05$). Left panel: home trade deficit as a share of home goods GDP (blue, declining from an initial peak toward steady state) and foreign trade surplus as a share of foreign goods GDP (red, rising toward steady state). Right panel: wage ratio of goods producers to service producers for the home economy (blue) and foreign economy (red); the dotted gray line marks the common closed-economy benchmark. In the open-economy steady state the home ratio lies below the autarky level (deindustrialization) while the foreign ratio lies above it.

3 Tariff Policy

Think of the home economy as the United States and the foreign economy as the rest of the world. The open-economy equilibrium just described features a structural trade deficit: the dollar is "overvalued" in the sense that it commands more real goods than it would if the U.S. were not the reserve-currency issuer,[3] and the tradables sector has contracted relative to the closed-economy benchmark. Both phenomena follow from the same source: foreign demand for U.S.-issued transaction assets. This is what Miran Miran (2024) identifies as the mechanism behind the persistent U.S. trade deficit.

The policy question is whether a unilateral import tariff can undo this. We model the tariff as a proportional tax θ on goods produced abroad and consumed at home. Given perfect substitutability, all trade flows from the foreign economy to the home economy; without loss of generality, foreign consumers consume only foreign-produced goods. Tariff revenue is rebated lump-sum to home old-age consumers.

Let p_t^k and p_t^{k*} be the prices of home- and foreign-produced goods. Let k_{Ht} and k_{Ft} denote home consumption of domestic and foreign goods, and k_{Ft}^* consumption of foreign goods in the foreign economy. In equilibrium $p_t^k = (1+\theta)\, p_t^{k*}$.

[3]Dollar "overvaluation" here takes the form of a lower price level since there is no nominal exchange rate.

Definition 3. *A tariff distorted competitive equilibrium is a sequence of*

domestic and foreign allocations and money holdings
$\left\{\left(c_t^i, k_{Ht}^i, k_{Ft}^i, m_t^i\right), \left(c_t^{i*}, k_{Ft}^{i*}, m_t^{i*}\right)\right\}_{t=0,\ i\in\{1,2,3\}}^{\infty}$,
mobile-labor allocations $\{n_t, n_t^*\}_{t=0}^{\infty}$,
prices $\left\{p_t^c, p_t^{c*}, p_t^k, p_t^{k*}\right\}_{t=0}^{\infty}$,
wages $\left\{w_t^i, w_t^{i*}\right\}_{t=0,\ i\in\{1,2,3\}}^{\infty}$,
and government policy $\{\theta, M_t, \tau_t\}_{t=0}^{\infty}$,

such that

1. *Given prices, wages and government policy, allocation* $\{c_t^i, k_t^i, m_t^i\}_{t=0}^{\infty}$ *solves the problem of domestic household* i*':* $\max u\left(c_{t+1}^i, k_{Ht+1}^i + k_{Ft+1}^i\right)$*, subject to,*

$$m_t^i \le w_t^i \text{ and } p_{t+1}^c c_{t+1}^i + p_{t+1}^k k_{Ht+1}^i + (1+\theta)\, p_{t+1}^{k*} k_{Ft+1}^i \le m_t^i + \frac{\tau_{t+1}}{3};$$

2. *Given prices, wages, and government policy, allocation* $\{c_t^{i*}, k_t^{i*}, m_t^{i*}\}_{t=0}^{\infty}$ *solves the problem of the foreign individual worker* i*' (note: the difference is that they do not receive the transfer):* $\max u\left(c_{t+1}^{i*}, k_{Ft+1}^{i*}\right)$ *subject to,*

$$m_t^{i*} \le w_t^{i*} \text{ and } p_{t+1}^{c*} c_{t+1}^{i*} + p_{t+1}^{k*} k_{Ft+1}^{i*} \le m_t^{i*}$$

3. *Domestic wages are determined according to equations (1), (2) and (3) (and similarly for foreign wages).*

4. *Government budget constraint holds:*

$$\tau_t = M_t - M_{t-1} + \theta p_t^k \left(\sum_{i=1,2,3} k_{Ft}^i\right)$$

5. *Markets clear:*

$$\sum_{i=1,2,3} c_t^i = (1-n_t)^\alpha$$
$$\sum_{i=1,2,3} c_t^{i*} = (1-n_t^*)^\alpha$$
$$\sum_{i=1,2,3} k_{Ht}^i = n_t^\alpha$$
$$\sum_{i=1,2,3} k_{Ft}^i + \sum_{i=1,2,3} k_{Ft}^{i*} = n_t^{*\alpha}$$
$$\sum_{i=1,2,3} m_t^i + \sum_{i=1,2,3} m_t^{i*} = M_t$$

The equilibrium conditions under the tariff are summarized in the following lemma.

Lemma 2. *Equilibrium allocation and prices in the open economy with tariff is characterized by the following system of equations for $t > 0$:*

$$(1-n_t^*)\, n_t^{*\alpha-1} = (1-\phi)\,\frac{p_{t-1}^{k*}}{p_t^{k*}} n_{t-1}^{*\alpha-1} \tag{16}$$

$$\begin{aligned}(1-n_t)\, n_t^{\alpha-1} = (1-\phi)\Bigg[&\mu\frac{p_{t-1}^k}{p_t^k} n_{t-1}^{\alpha-1} + (\mu-1)\,\frac{p_{t-1}^{k*}}{p_t^k} n_{t-1}^{*\alpha-1} \\ &+ \theta\frac{p_t^{k*}}{p_t^k}\left(n_t^{*\alpha} - \phi\frac{p_{t-1}^{k*}}{p_t^{k*}} n_{t-1}^{*\alpha-1}\right)\Bigg]\end{aligned} \tag{17}$$

$$\begin{aligned}n_t^\alpha + n_t^{*\alpha} = \phi\Bigg[&\mu\frac{p_{t-1}^k}{p_t^k} n_{t-1}^{\alpha-1} + (\mu-1)\,\frac{p_{t-1}^{k*}}{p_t^k} n_{t-1}^{*\alpha-1} \\ &+ \theta\frac{p_t^{k*}}{p_t^k}\left(n_t^{*\alpha} - \phi\frac{p_{t-1}^{k*}}{p_t^{k*}} n_{t-1}^{*\alpha-1}\right)\Bigg] + \phi\frac{p_{t-1}^{k*}}{p_t^{k*}} n_{t-1}^{*\alpha-1}\end{aligned} \tag{18}$$

and

$$p_t^k = (1+\theta) p_t^{k*}$$

The key observation is equation (16): it is identical to the no-tariff condition (6). The tariff does not appear in the foreign equilibrium condition at all. This is not a coincidence. The foreign economy's decisions—how much to produce, how much to export, how much money to hold—are governed entirely by its demand for the home transaction asset. None of these margins depend on what the home country charges at the border. As a result, θ drops out of the foreign steady state entirely.

In steady state, imposing the steady-state condition in equation (16) gives

$$n^* = \frac{\mu - 1}{\mu} + \frac{\phi}{\mu}.$$

This is independent of the tariff rate. Imposing a tariff on imports does not affect the foreign steady-state allocation.

Turning to the home economy: imposing the steady-state condition and substituting $p_t^k = (1+\theta)p_t^{k*}$ in equation (17) and simplifying (substituting for n^* and rearranging) yields

$$\phi n^{\alpha-1} - n^{\alpha} = (1-\phi)\left(\frac{\mu-1}{\mu}\right) n^{*\alpha-1}$$

This too is independent of θ. The home labor allocation is also invariant to the tariff. Long-run sectoral employment and output are unchanged.

Let $K_F \equiv \sum_{i=1,2,3} k_{Ft}^i$ denote aggregate home imports of foreign-produced goods.

Proposition 2 (Long-Run Tariff Neutrality). *Suppose $\mu > 1$. For any tariff rate $\theta \geq 0$, the steady-state allocation (n, n^*, K_F) is invariant to θ. In particular:*

(i) $n^* = 1 - (1-\phi)/\mu$*, independently of* θ*;*

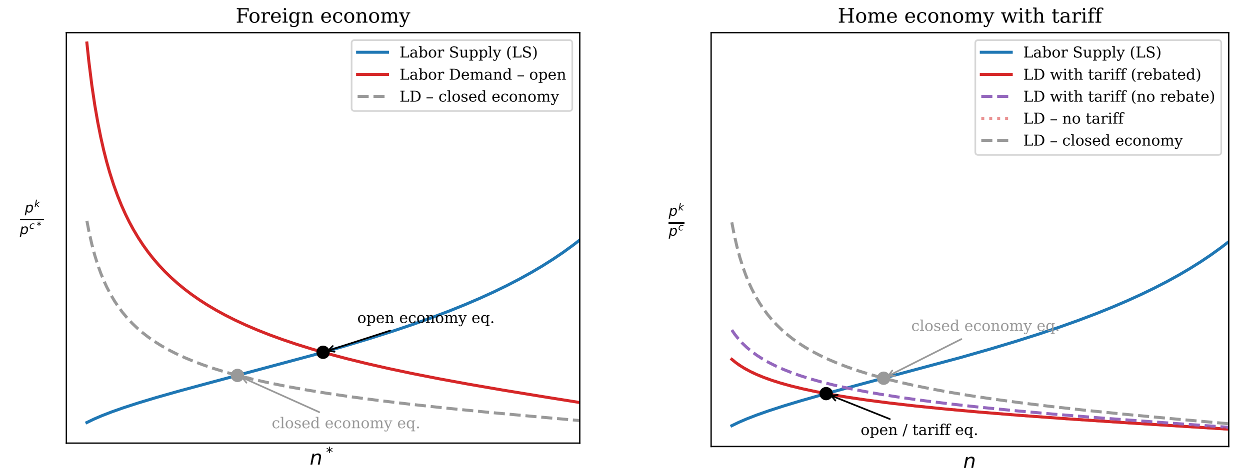
(ii) n *satisfies* $\phi n^{\alpha}/n - n^{\alpha} = (1-\phi)\big[(\mu-1)/\mu\big]F(n^*)/n^*$*, independently of* θ*;*

(iii) $K_F = \big[(\mu-1)/\mu\big]n^{*\,\alpha-1}$*, independently of* θ*.*

Proof. See Appendix A.4. □

Why is the tariff neutral? The intuition is straightforward. The foreign economy must export tradables to acquire home money; the quantity it exports is determined by its need for the transaction asset, not by the tariff wedge. Formally, foreign steady-state behavior is governed solely by the service-market clearing condition (16), which does not contain θ. This pins down n^* (and hence K_F) independently of the tariff. At home, the tariff raises the consumer price of imported goods relative to the producer price, but—because tariff revenue is rebated lump-sum to home old-age consumers—the extra spending power exactly offsets the price increase. The home labor market therefore sees no change in the real return to tradables production, leaving n unchanged.

In short: the tariff cannot eliminate the trade deficit because the deficit is the equilibrium counterpart of foreign demand for the home transaction asset, not an outcome of distorted prices.



(a) Foreign economy.

(b) Home economy.

Figure 4: Steady-state effects of an import tariff. Colors: the blue curve is labor supply (sectoral choice), the red curve is labor demand, and the dashed gray curve is the closed-economy labor demand with the gray dot denoting the closed-economy equilibrium. In the home panel, the purple dashed curve is labor demand in the tariff economy without the lump-sum tariff rebate. Despite shifts in home labor demand, the equilibrium mobile labor allocation n is unchanged relative to the no-tariff open-economy benchmark.

3.1 A labor supply–labor demand view of tariffs

In steady state, the foreign economy is unaffected by the home import tariff: the foreign labor supply schedule is unchanged and the foreign labor demand condition continues to pin down n^* as in Proposition 1. In the home economy, the tariff modifies the relationship between relative prices and expenditure shares (because tariff revenue is rebated lump-sum to home old-age consumers). Combining steady-state service demand with $p^k = (1+\theta)p^{k*}$ yields the home "labor demand" schedule under a tariff:

$$\frac{p^k}{p^c} = \frac{\phi}{1-\phi}\,\frac{(1-n)^\alpha}{\left(1-\frac{1}{\mu}\right)\frac{1}{1+\theta}\,n^{*\,\alpha-1} + \frac{\theta}{1+\theta}\left(n^{*\,\alpha} - \frac{\phi}{\mu}n^{*\,\alpha-1}\right) + n^\alpha} \qquad \text{(steady state).} \tag{19}$$

Figure 4 illustrates the key point: although the tariff shifts the home labor-demand schedule, the intersection with the unchanged labor-supply schedule occurs at the same n. The tariff therefore does not "reindustrialize" the home economy in the long run.

3.2 Valuation Effects and Trade-Deficit Measurement

What the tariff does change is relative prices. Home-produced goods are always more expensive than foreign-produced goods by the factor $(1+\theta)$. A higher tariff therefore compresses the foreign producer price relative to the home consumer price. Since import quantities are unchanged, this price wedge is the only channel through which the measured trade deficit can move.

Denote the aggregate demand for goods in the foreign country by K_{Ft}. Then

$$K_{Ft} = \phi \frac{p_{t-1}^{k*}}{p_t^{k*}} n_{t-1}^{*\alpha-1}$$

and the amount of goods imported by the home economy is

$$K_{Ft} = n_t^{*\alpha} - K_{Ft}^* = n_t^{*\alpha} - \phi \frac{p_{t-1}^{k*}}{p_t^{k*}} n_{t-1}^{*\alpha-1}$$

The trade deficit as a share of GDP is

$$\begin{aligned} \frac{\text{EXP}_t}{\text{GDP}_t} - 1 &= \frac{p_t^{k*} K_{Ft}}{p_t^k n_t^\alpha + p_t^c (1-n_t)^\alpha} \\ &= \frac{p_t^{k*}}{p_t^k} \frac{n_t^{*\alpha} - \phi \frac{p_{t-1}^{k*}}{p_t^{k*}} n_{t-1}^{*\alpha-1}}{\frac{n_t^\alpha}{n_t}} \end{aligned}$$

In steady state (after substituting for n^*):

$$\frac{\text{EXP}_t}{\text{GDP}_t} - 1 = \frac{1}{1+\theta} \left(\frac{\mu-1}{\mu} \right) n^{*\alpha-1} \frac{n}{n^\alpha}. \tag{20}$$

The measured trade deficit falls as the tariff rises. But the decline is entirely mechanical. The import quantity K_F is pinned down by foreign production and demand—by μ, ϕ, and α—and is invariant to θ. What changes is the price at which imports enter the trade-balance calculation: a higher tariff depresses $p^{k*}/p^k = 1/(1+\theta)$, and since imports are valued at world prices while home GDP is valued at domestic prices, the ratio falls. This is a valuation effect, not a quantity effect. The home economy is not importing less; it is simply valuing its imports more cheaply relative to its own output.

3.3 Burden Sharing and Incidence

The long-run picture is this: quantities are unchanged, but prices are not. The tariff drives a wedge between what home consumers pay and what foreign producers receive. Both sides of this wedge are borne by someone.

Foreign exporters receive lower net-of-tariff prices—a terms-of-trade deterioration. Home consumers pay more for imported tradables. The home government collects the difference. In this sense the tariff looks more like an international income transfer than a border protection device: it shifts resources from foreign producers and home consumers to the home fiscal authority, without altering the real quantities that cross the border.

To see why incidence is shared, note that in equilibrium $p^k = (1+\theta)p^{k*}$. A higher tariff either raises the home consumer price p^k, compresses the foreign producer price p^{k*}, or some combination of both. In our model, both adjust, so both home consumers and foreign producers bear part of the wedge. This shared-incidence pattern aligns with the empirical findings in Amiti et al. (2019) and Fajgelbaum et al. (2020), and with the broader message on valuation channels in Caliendo et al. (2025) and Itskhoki and Mukhin (2025).

4 Extensions

The benchmark results rest on two simplifications worth relaxing. First, we assumed home and foreign tradables are perfect substitutes, so demand elasticity played no role. Second, we compared tariffs to the counterfactual of no policy; the natural alternative is a direct production subsidy to the tradables sector. We take up each in turn, and then examine how the fiscal disposition of tariff revenue affects the neutrality result.

4.1 Imperfect substitution between home and foreign tradables

Consider the following preferences in the home economy:

$$\nu\left(k_{Ht}, k_{Ft}, c_t\right) = \left[(1-\eta)^{\frac{1}{\rho}} k_{Ht}^{\frac{\rho-1}{\rho}} + \eta^{\frac{1}{\rho}} k_{Ft}^{\frac{\rho-1}{\rho}}\right]^{\frac{\phi\rho}{\rho-1}} c_t^{1-\phi}, \quad 0<\rho<\infty.$$

where k_{Ht} and k_{Ft} are demands for domestic and foreign-produced tradables in the domestic economy and ρ is the substitution elasticity. The limit $\rho \to \infty$ corresponds to our baseline (perfect substitutes).

For the foreign economy, assume

$$\nu\left(k_{Ft}^{*}, c_t^{*}\right) = k_{Ft}^{*\phi} c_t^{*1-\phi}.$$

As before, foreign consumers consume only foreign-produced tradables.

Home CES demand system: full derivation

Fix a date $t+1$ and suppress time subscripts where convenient. Home old-age tradables consumption is a composite of home and foreign varieties,

$$K \equiv \left[(1-\eta)^{\frac{1}{\rho}} K_H^{\frac{\rho-1}{\rho}} + \eta^{\frac{1}{\rho}} K_F^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}},$$

and period utility over the goods composite and services is

$$U = K^{\phi} c^{1-\phi}.$$

Given composite expenditure on tradables, $E_K \equiv p^k K_H + (1+\theta) p^{k*} K_F$, cost minimization implies the standard CES price index

$$P^k \equiv \left[(1-\eta)\left(p^k\right)^{1-\rho} + \eta\left((1+\theta)p^{k*}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}, \tag{21}$$

so that $E_K = P^k K$. The associated Hicksian (and, with homothetic preferences, Marshallian) demands are

$$K_H = (1-\eta)\left(\frac{p^k}{P^k}\right)^{-\rho} K, \tag{22}$$

$$K_F = \eta\left(\frac{(1+\theta)p^{k*}}{P^k}\right)^{-\rho} K. \tag{23}$$

Equivalently, the expenditure shares are

$$\frac{p^k K_H}{E_K} = \frac{(1-\eta)\left(p^k\right)^{1-\rho}}{(1-\eta)\left(p^k\right)^{1-\rho} + \eta\left((1+\theta)p^{k*}\right)^{1-\rho}}, \tag{24}$$

$$\frac{(1+\theta)p^{k*} K_F}{E_K} = \frac{\eta\left((1+\theta)p^{k*}\right)^{1-\rho}}{(1-\eta)\left(p^k\right)^{1-\rho} + \eta\left((1+\theta)p^{k*}\right)^{1-\rho}}. \tag{25}$$

Finally, Cobb–Douglas between K and c implies $E_K = \phi E$ and $p^c c = (1-\phi)E$, where E denotes total nominal expenditure by the home old at date $t+1$.

The demand in the foreign economy is

$$p^{c*}_{t+1} c^*_{t+1} = (1-\phi)\, p^{k*}_t n_t^{*\alpha-1} \tag{26}$$

$$p^{k*}_{t+1} K^*_{Ft+1} = \phi p^{k*}_t n_t^{*\alpha-1} \tag{27}$$

with optimal sector choice

$$\frac{p_t^{c*}}{p_t^{k*}} = \left(\frac{n_t^*}{1-n_t^*}\right)^{\alpha-1}. \tag{28}$$

The demand in the home economy is

$$p_{t+1}^c c_{t+1} = (1-\phi)\left(\mu p_t^k n_t^{\alpha-1} + (\mu-1)\, p_t^{k*} n_t^{*\alpha-1} + \theta p_{t+1}^{k*} k_{Ft+1}\right) \tag{29}$$

$$p_{t+1}^k K_{Ht+1} = \frac{1-\eta}{1-\eta+\eta\left((1+\theta)\frac{p_{t+1}^{k*}}{p_{t+1}^k}\right)^{1-\rho}} \phi\left(\mu p_t^k n_t^{\alpha-1} + (\mu-1)\, p_t^{k*} n_t^{*\alpha-1} + \theta p_{t+1}^{k*} K_{Ft+1}\right) \tag{30}$$

$$(1+\theta)\, p_{t+1}^{k*} K_{Ft+1} = \frac{\eta\left((1+\theta)\frac{p_{t+1}^{k*}}{p_{t+1}^k}\right)^{1-\rho}}{1-\eta+\eta\left((1+\theta)\frac{p_{t+1}^{k*}}{p_{t+1}^k}\right)^{1-\rho}} \phi\left(\mu p_t^k n_t^{\alpha-1} + (\mu-1)\, p_t^{k*} n_t^{*\alpha-1} + \theta p_{t+1}^{k*} K_{Ft+1}\right) \tag{31}$$

with optimal sector choice

$$\frac{p_t^c}{p_t^k} = \left(\frac{n_t}{1-n_t}\right)^{\alpha-1}. \tag{32}$$

Imposing steady state in the foreign economy yields

$$n^* = \frac{\mu-1}{\mu} + \frac{\phi}{\mu} \tag{33}$$

$$k_F^* = \frac{\phi}{\mu} n^{*\alpha-1}. \tag{34}$$

Using market clearing for foreign-produced tradables, steady-state home imports are

$$K_F = n^{*\alpha} - \frac{\phi}{\mu} n^{*\alpha-1} = \left(\frac{\mu-1}{\mu}\right) n^{*\alpha-1}. \tag{35}$$

As in the benchmark, the long-run import quantity is pinned down by foreign production and foreign demand, invariant to the tariff.

In steady state, the home allocation and the relative price p^k/p^{k*} solve a system of two equations derived from goods-market clearing and the CES demand system:

$$\phi n^{\alpha-1} - n^\alpha = (1-\phi)(1+\theta)\frac{p^{k*}}{p^k} n^{*\alpha-1}\left(\frac{\mu-1}{\mu}\right) \tag{36}$$

$$n = \frac{(1-\eta)\,\phi}{(1-\phi)\,\eta\left((1+\theta)\frac{p^{k*}}{p^k}\right)^{1-\rho} + (1-\eta)}. \tag{37}$$

Eliminating $(1+\theta)\,\frac{p^{k*}}{p^k}$ between (36) and (37) shows that the steady-state labor allocation n is independent of θ. Long-run sectoral allocation and outputs are invariant to the tariff even when tradables are imperfect substitutes.

Proposition 3 (CES Tariff Neutrality)**.** *Under one-way CES demand (home consumers substitute between home and foreign tradables with elasticity $\rho > 0$; foreign consumers are specialized in the foreign variety), the steady-state labor allocations (n, n^*) and the import quantity K_F are invariant to the tariff rate θ. The measured trade-deficit-to-GDP ratio falls with θ through the relative price p^{k*}/p^k, not through any change in quantities.*

Proof. See Appendix A.5. □

The trade deficit as a share of home GDP in steady state is

$$\frac{p^{k*}K_F}{p^k n^\alpha + p^c\,(1-n)^\alpha} = \frac{p^{k*}}{p^k}\frac{n^{*\alpha-1}}{n^{\alpha-1}}\left(\frac{\mu-1}{\mu}\right), \tag{38}$$

which is decreasing in θ through the price ratio p^{k*}/p^k. As in the benchmark, the measured deficit falls through valuation rather than through quantity adjustment.

The one-way CES extension above treats the foreign economy as specialized in its own variety. A more symmetric environment allows both countries to substitute between home and foreign tradables (an Armington framework). In that setting the reserve-currency mechanism still pins down the *net* real resource transfer, while tariffs can meaningfully compress *gross* trade flows and may further shift home production toward non-tradables. The formal analysis, including the derivation of equilibrium gross trade flows, the terms-of-trade response to tariffs, and the finding that home tradables employment can *fall* with the tariff (further deindustrialization), is developed in Appendix C.

4.2 Domestic Industrial Policy

Suppose that, instead of imposing a tariff, the home country subsidizes production in the goods-producing (tradables) sector, financed by a lump-sum tax. The decision problem in the foreign country is unchanged.

Home producers in the goods-producing sector receive a subsidy κ per unit of output sold. Since the subsidy is paid to producers rather than levied at the border, the law of one price continues to hold for consumer prices: $p_t^k = p_t^{k*}$. Therefore,

$$w_t^1 = (1+\kappa)\, p_t^k\, (1-\alpha)\, n_t^\alpha$$
$$w_t^2 = p_t^c\, (1-\alpha)\, (1-n_t)^\alpha$$
$$w_t^3 = (1+\kappa)\, p_t^k \alpha n_t^{\alpha-1} = p_t^c \alpha\, (1-n_t)^{\alpha-1}$$

Therefore,

$$\frac{p_t^c}{p_t^k} = (1+\kappa)\, n_t^{\alpha-1} \frac{1-n_t}{(1-n_t)^\alpha} = (1+\kappa)\, \frac{n_t^{\alpha-1}}{(1-n_t)^{\alpha-1}} \tag{39}$$

The government budget constraint in the home economy is

$$M_t - M_{t-1} - \kappa p_t^k n_t^\alpha = \tau_t. \tag{40}$$

The demand in the home economy is

$$p_{t+1}^c c_{t+1} = (1-\phi)\left(\mu\,(1+\kappa)\, p_t^k n_t^{\alpha-1} + (\mu-1)\, p_t^{k*} n_t^{*\alpha-1} - \kappa p_{t+1}^k n_{t+1}^\alpha\right) \tag{41}$$

$$p_{t+1}^k K_{Ht+1} + p_{t+1}^{k*} K_{Ft+1} = \phi\left(\mu\,(1+\kappa)\, p_t^k n_t^{\alpha-1} + (\mu-1)\, p_t^{k*} n_t^{*\alpha-1} - \kappa p_{t+1}^k n_{t+1}^\alpha\right) \tag{42}$$

and in the foreign economy

$$p_{t+1}^{c*} c_{t+1}^* = (1-\phi)\, p_t^{k*} n_t^{*\alpha-1} \tag{43}$$

$$p_{t+1}^{k*} K_{Ft+1}^* = \phi p_t^{k*} n_t^{*\alpha-1} \tag{44}$$

along with optimal sector choice condition (28).

Imposing steady state in equation (43) and using optimal sectoral condition (28):

$$n^* = \frac{\mu-1}{\mu} + \frac{\phi}{\mu}$$
$$k_F^* = \frac{\phi}{\mu} n^{*\alpha-1}$$

and from market clearing, imports in the home country are

$$K_F = n^{*\alpha} - \frac{\phi}{\mu} n^{*\alpha-1} = \left(\frac{\mu-1}{\mu}\right) n^{*\alpha-1}$$

Rewriting equation (41) in steady state and eliminating relative prices using equation (39):

$$(1+\kappa)\left(n^{\alpha-1} - n^\alpha\right) = (1-\phi)\left((1+\kappa)\, n^{\alpha-1} + \left(\frac{\mu-1}{\mu}\right) n^{*\alpha-1} - \kappa n^\alpha\right)$$

which yields

$$(1+\kappa)\,\phi n^{\alpha-1} - (1+\phi\kappa)\,n^{\alpha} = (1-\phi)\left(\frac{\mu-1}{\mu}\right) n^{*\alpha-1}.$$

Unlike in the benchmark and the tariff case, the home labor allocation depends on κ: as subsidies increase, employment and output in the goods-producing sector increase.

Proposition 4 (Industrial Policy)**.** *Suppose $\kappa \geq 0$ is a production subsidy to home tradables. In steady state:*

(i) The foreign labor allocation n^* and the import quantity K_F are invariant to κ.

(ii) The home tradables labor share n is strictly increasing in κ.

Proof. See Appendix A.6. □

Why does the production subsidy succeed where the tariff fails? The key difference is where the policy instrument acts. A tariff acts on the *border price of the imported good* and, because the revenue is rebated, has no net effect on the relative return to home tradables production. A production subsidy, by contrast, directly raises the *marginal revenue product of labor in the home goods sector*, shifting the labor supply curve for mobile workers in favor of tradables. Foreign workers face exactly the same conditions as before, so n^* and K_F are unaffected. The takeaway is that the channel through which policy raises n matters: demand-side instruments (tariffs) that merely redistribute income between sectors cannot achieve what supply-side instruments (subsidies) that alter the private return to production can.

Graphical interpretation. The subsidy directly raises the marginal revenue product of labor in the home goods sector. In the labor-supply/labor-demand diagram, this shows up as a shift in the home labor supply schedule (sectoral choice) for the mobile factor, while the demand schedule is unchanged. Figure 5 illustrates how a production subsidy raises the steady-state mobile labor allocation n in the home tradables sector.

The subsidy does not affect the quantity of goods imported. But because the relative wage in the goods sector rises, home goods production increases and home services production falls. The trade deficit as a share of GDP is

$$\begin{aligned}\frac{p_t^{k*} K_{Ft}}{p_t^k n_t^{\alpha} + p_t^c (1-n_t)^{\alpha}} &= \frac{p_t^{k*}}{p_t^k} \frac{\left(\frac{\mu-1}{\mu}\right) n^{*\alpha-1}}{n^{\alpha} + \frac{p_t^c}{p_t^k}(1-n_t)^{\alpha}} \\ &= \frac{\left(\frac{\mu-1}{\mu}\right) n^{*\alpha-1}}{(1+\kappa)\,n^{\alpha-1} - \kappa n^{\alpha}}\end{aligned}$$

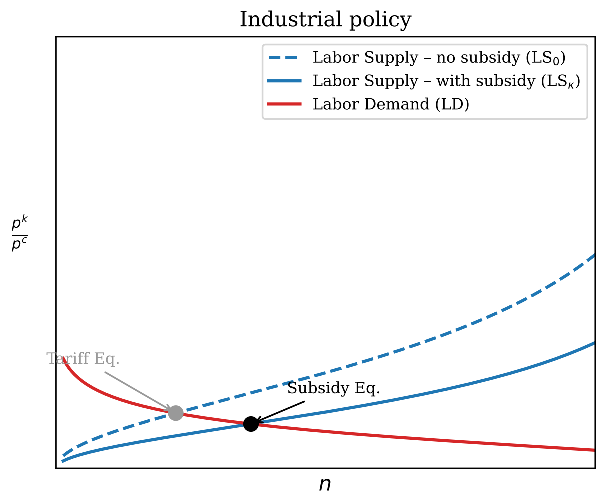


Figure 5: Home-economy steady state with a production subsidy to the goods sector. Colors: the red curve is labor demand, the solid blue curve is labor supply with the subsidy, and the dashed blue curve is labor supply without the subsidy; the gray dot denotes the equilibrium without the subsidy. The subsidy shifts the home labor-supply schedule up, raising the equilibrium mobile labor share in the goods sector n relative to the no-subsidy open-economy benchmark.

The denominator can be rewritten as

$$(1+\kappa)\, n^{\alpha-1} - \kappa n^{\alpha} = \left(\frac{1-\phi}{\phi}\right)\left(\frac{\mu-1}{\mu}\right) n^{*\alpha-1} + \frac{n^{\alpha}}{\phi}$$

which is increasing in κ. So, as with the tariff, the subsidy reduces the trade-deficit-to-GDP ratio, despite having no effect on the import quantity. The difference is that under the subsidy, more goods and fewer services are produced at home, and households consume relatively more goods than in the benchmark.

The point is simple but important. Tariffs and production subsidies are both government interventions in the tradables sector, but they operate on different margins. If the goal is reindustrialization, the instrument has to reach the right margin: the relative return to producing tradables at home.

4.3 Fiscal Disposition of Tariff Revenue

The benchmark analysis assumes tariff revenue is rebated lump-sum to home old-age consumers, leaving μ exogenous. Here we consider two alternative fiscal arrangements in which the home government has an exogenous expenditure requirement $g \in (0, 1)$: it must consume a fraction g of nominal GDP each period.

Environment. The government has two budget constraints, one for money issuance and one for tariff revenue:

$$\text{(GBC-money)} \qquad M_t - M_{t-1} = \text{money-financed expenditure},$$

$$\text{(GBC-tariff)} \qquad \theta\, p_t^{k*}\, K_{Ft} = \text{tariff revenue}.$$

How these two sources are combined to meet the total expenditure $g \cdot \text{GDP}_t$ defines the two cases below. The full formal definitions, equilibrium conditions, and steady-state derivations are in Appendix B. We state the main results here.

Tariff neutrality survives in one case (Case A) but breaks down in another (Case B), where higher tariffs raise the inflation rate and generate non-trivial reallocation effects.

Case A: Tariff revenue used to finance government spending (Proposition 5)

Suppose the home government spends a fraction g of nominal GDP and finances expenditure with seigniorage *and* tariff revenue combined (government budget constraint (50)). The money-growth rate μ is endogenous, but the tariff enters the government budget constraint in a way that leaves the steady-state conditions for n and n^* unaffected.

Proposition 5 (Case A: Tariff Neutrality with Fiscal Spending)**.** *In Case A, the steady-state labor allocations n and n^* are invariant to the tariff rate θ. There exists a unique inflation rate $\mu \in \left(1, \frac{1}{1-g}\right)$ satisfying the government budget constraint, independently of θ.*

Proof. See Appendix A.7. □

After imposing steady state, n and n^* are pinned down solely by μ (as in the benchmark), and the budget constraint determines μ independently of θ.

Case B: Tariff revenue rebated to old consumers, spending via seigniorage (Proposition 6)

Now suppose the government funds its spending g entirely via money creation (budget constraint (58)), while tariff revenue is separately rebated lump-sum to home old-age consumers (budget constraint (59)). The rebate enriches home consumers and affects their goods demand, which in turn requires a different μ to clear the government's budget. This linkage between θ and μ breaks the neutrality.

Proposition 6 (Case B: Endogenous Inflation and Non-Monotone Reallocation)**.** *In Case B, for any $\theta \geq 0$:*

(i) $n^*(\theta) = 1 - (1-\phi)/\mu(\theta)$ *is strictly increasing in* θ*.*

(ii) $\mu(\theta)$ *is strictly increasing in* θ*:* $\mu'(\theta) > 0$*.*

(iii) Home labor allocation is

$$n(\theta) = 1 - \frac{1-\phi}{1-g}\left(\frac{1+\theta}{\mu(\theta)} - \theta(1-g)\right),$$

whose sign of $dn/d\theta$ *is ambiguous.*

(iv) As $\theta \to \infty$*,* $\mu(\theta) \to \frac{1}{1-g}$ *and* $n(\theta) \to \phi$*.*

(v) For sufficiently small g*,* $n(\theta)$ *is non-monotone: it initially decreases in* θ *(from* $n(0) > \phi$ *down toward* ϕ*) before the limit* $n(\infty) = \phi$ *is approached from above.*

Proof. See Appendix A.8. □

The economic intuition for part (i) is that higher tariffs raise the home consumer's effective income (via the rebate), which bids up aggregate demand and requires faster money growth μ to finance the government's fixed expenditure. This faster inflation reduces the real value of foreign money holdings, pushing foreign workers further into tradables: n^* rises. At home, the direct expenditure effect of the rebate raises goods demand and draws labor into tradables, but higher inflation simultaneously erodes real purchasing power and can dominate for moderate θ, producing a non-monotone response in n.

5 Conclusion

Start from a simple observation: if the U.S. trade deficit is the equilibrium counterpart of foreign demand for dollar-denominated assets, then the deficit cannot be eliminated by making imports more expensive. The foreign economy exports tradables not because it wants to; it does so because it needs dollars. A tariff does not change that need.

We have built a model to make this argument precise. In equilibrium, the foreign labor allocation and the import quantity are governed entirely by the foreign economy's demand for the home transaction asset—both are independent of the tariff rate. At home, the tariff raises import prices, but when revenue is rebated to consumers their spending power is restored. The home labor market sees no change in the relative return to tradables production. Sectoral employment and output are invariant.

What does change is relative prices. A higher tariff compresses the foreign producer price relative to the home consumer price, reducing the measured trade-deficit-to-GDP ratio through valuation rather than through quantity adjustment. The burden is shared: foreign exporters receive less per unit, home consumers pay more per unit, and the home government collects the difference.

Three extensions refine this message. Imperfect substitutability between home and foreign tradables (CES demand) does not restore tariff effectiveness: long-run quantities remain invariant, and only the valuation channel operates (Proposition 3). A production subsidy to home tradables does raise steady-state employment in the goods sector, because it operates directly on the margin that tariffs miss: the relative return to producing tradables (Proposition 4). The fiscal disposition of tariff revenue matters for whether neutrality survives: if revenue finances a fixed government expenditure, neutrality holds (Proposition 5); if revenue is rebated while government spending is financed through money creation, the resulting inflation channel breaks neutrality in a non-monotone way (Proposition 6).

The broader lesson is about the interaction between trade policy and the monetary system. In a reserve-currency economy, the real trade balance is determined by asset demand, not by trade policy. Tariffs can redistribute income between countries and raise domestic revenue; they cannot rebalance trade or reindustrialize the economy on their own. A policy aimed at reducing the trade deficit in real terms would need to address the underlying demand for U.S.-issued assets—a much harder problem than adjusting the tariff schedule.

Several important dimensions are left for future work. The model abstracts from aggregate demand, nominal rigidities, and short-run exchange-rate dynamics, all of which may give tariffs non-trivial transitional effects even when long-run allocations are neutral. We also set aside welfare analysis: while higher tariffs raise home fiscal revenue and shift the terms of trade in some scenarios, the optimal tariff problem in a reserve-currency setting—balancing these gains against the costs of higher domestic consumer prices and the risk of eroding reserve-currency status—remains an open question. Finally, introducing a discrete erosion of reserve-currency status as a response to high tariffs would add a strategic dimension not captured here. These extensions are left for future research.

References

Amiti, M., S. J. Redding, and D. E. Weinstein (2019). The impact of the 2018 tariffs on prices and welfare. *Journal of Economic Perspectives 33*(4), 187–210.

Autor, D. H., D. Dorn, and G. H. Hanson (2013). The china syndrome: Local labor market effects of import competition in the United States. *American Economic Review 103*(6), 2121–2168.

Bagwell, K. and R. W. Staiger (1999). An economic theory of GATT. *American Economic Review 89*(1), 215–248.

Bagwell, K. and R. W. Staiger (2002). *The Economics of the World Trading System.* Cambridge, MA: The MIT Press.

Bernanke, B. S. (2005). The global saving glut and the U.S. current account deficit. Remarks at the Sandridge Lecture, Virginia Association of Economists, Richmond, Virginia, March 10, 2005.

Caballero, R. J., E. Farhi, and P.-O. Gourinchas (2016, February). Safe asset scarcity and aggregate demand. NBER Working Paper 22044, National Bureau of Economic Research.

Caballero, R. J., E. Farhi, and P.-O. Gourinchas (2017). The safe assets shortage conundrum. *Journal of Economic Perspectives 31*(3), 29–46.

Caliendo, L., S. S. Kortum, and F. Parro (2025, July). Tariffs and trade deficits. NBER Working Paper 34003, National Bureau of Economic Research.

Fajgelbaum, P. D., P. K. Goldberg, P. J. Kennedy, and A. K. Khandelwal (2020). The return to protectionism. *Quarterly Journal of Economics 135*(1), 1–55.

Farhi, E. and M. Maggiori (2018). A model of the international monetary system. *The Quarterly Journal of Economics 133*(1), 295–355.

Gopinath, G., E. Boz, C. Casas, F. J. Díez, P.-O. Gourinchas, and M. Plagborg-Møller (2020). Dominant currency paradigm. *American Economic Review 110*(3), 677–719.

Gourinchas, P.-O. and H. Rey (2005, August). From world banker to world venture capitalist: US external adjustment and the exorbitant privilege. NBER Working Paper 11563, National Bureau of Economic Research.

Gourinchas, P.-O. and H. Rey (2013, July). External adjustment, global imbalances and valuation effects. NBER Working Paper 19240, National Bureau of Economic Research.

Itskhoki, O. and D. Mukhin (2025, May). The optimal macro tariff. NBER Working Paper 33839, National Bureau of Economic Research.

Jones, R. W. (1971). A three-factor model in theory, trade, and history. In J. N. Bhagwati, R. W. Jones, R. A. Mundell, and J. Vanek (Eds.), *Trade, Balance of Payments and Growth: Papers in International Economics in Honor of Charles P. Kindleberger*, pp. 3–21. Amsterdam: North-Holland.

Krishnamurthy, A. and A. Vissing-Jorgensen (2012). The aggregate demand for treasury debt. *Journal of Political Economy 120*(2), 233–267.

Lerner, A. P. (1936). The symmetry between import and export taxes. *Economica 3*(11), 306–313.

Metzler, L. A. (1949). Tariffs, the terms of trade, and the distribution of national income. *Journal of Political Economy 57*(1), 1–29.

Miran, S. (2024). A user's guide to restructuring the global trading system. *Hudson Bay Capital, November*.

Obstfeld, M. and K. Rogoff (2005). Global current account imbalances and exchange rate adjustments. *Brookings Papers on Economic Activity 2005*(1), 67–123.

A Proofs

A.1 Proof of Lemma 1

Start from the demand for service in foreign country

$$p_t^{c*} \sum_{i=1,2,3} c_t^{i*} = (1-\phi) \sum_{i=1,2,3} w_{t-1}^{i*}$$

On the left-hand side, total consumption of service equals total production. On the right-hand side, total income in the foreign economy equals total value of output. Therefore,

$$p_t^{c*} F(1-n_t^*) = (1-\phi)\left(p_{t-1}^k n_{t-1}^{*\alpha} + p_{t-1}^{c*}\left(1-n_{t-1}^*\right)^{\alpha}\right)$$

The optimal choice by the type 3 worker implies

$$\frac{p_t^k}{p_t^{c*}} = \frac{n_t^*}{1-n_t^*} \frac{\left(1-n_t^*\right)^{\alpha}}{n_t^{*\alpha}}$$

Using this to simplify gives equation (7):

$$\left(1-n_t^*\right) n_t^{*\alpha-1} = (1-\phi) \frac{p_{t-1}^k}{p_t^k} n_{t-1}^{*\alpha-1}$$

Similarly, demand for goods in the foreign economy is

$$\sum_{i=1,2,3} k_t^{i*} = \phi \frac{p_{t-1}^k}{p_t^k} n_{t-1}^{*\alpha-1}$$

The demand for service in the home economy is

$$p_t^c \sum_{i=1,2,3} c_t^i = (1-\phi)(\sum_{i=1,2,3} w_{t-1}^i + (\mu - 1)M_{t-1})$$

where

$$\sum_{i=1,2,3} w_{t-1}^i = p_{t-1}^k n_{t-1}^\alpha + p_{t-1}^c \left(1 - n_{t-1}\right)^\alpha$$

and

$$M_{t-1} = p_{t-1}^k \left(n_{t-1}^\alpha + n_{t-1}^{*\alpha}\right) + p_{t-1}^{c*} \left(1 - n_{t-1}^*\right)^\alpha + p_{t-1}^c \left(1 - n_{t-1}\right)^\alpha$$

The first equation is the national income accounting identity. The second is the total demand for money by the young generation of cohort $t-1$. The optimal sector choice of type 3 at home implies

$$\frac{p_t^k}{p_t^c} = \frac{n_t}{1-n_t} \frac{\left(1-n_t\right)^\alpha}{n_t^\alpha}$$

Using these equations to simplify service demand at home gives equation (6):

$$\left(1 - n_t\right) n_t^{\alpha - 1} = \left(1 - \phi\right) \frac{p_{t-1}^k}{p_t^k} \left(\mu n_{t-1}^{\alpha-1} + \left(\mu - 1\right) n_{t-1}^{*\alpha-1}\right)$$

Similarly, demand for goods in the home economy is

$$k_t = \phi \frac{p_{t-1}^k}{p_t^k} \left(\mu n_{t-1}^{\alpha-1} + \left(\mu - 1\right) n_{t-1}^{*\alpha-1}\right)$$

Adding demand for goods in home and foreign economies and using goods-market clearing gives equation (8):

$$n_t^\alpha + n_t^{*\alpha} = \phi \mu \frac{p_{t-1}^k}{p_t^k} \left(n_{t-1}^{\alpha-1} + n_{t-1}^{*\alpha-1}\right)$$

The initial old in the home economy hold the initial stock of money M_0 and can buy goods and services. Service demand at home together with service-market clearing implies equation (10):

$$p_0^c \left(1 - n_0\right)^\alpha = \left(1 - \phi\right) M_0.$$

The initial old in the foreign economy have no money and hence no consumption. Since foreign services are non-tradable and no one can pay for them, no services are produced. All labor goes to goods production, which is shipped to the home economy in exchange for money. Together with optimal goods demand in the domestic economy this implies equation (11):

$$p_0^k \left(n_0^{*\alpha} + n_0^{\alpha}\right) = \phi M_0.$$

Finally, equation (12) is the optimal sector choice for type 3 households in the home economy.

A.2 Proof of Proposition 1

Imposing a steady-state condition in (7) we get

$$1 - n^* = \frac{1-\phi}{\mu}$$

and therefore

$$n^* = 1 - \frac{1-\phi}{\mu}.$$

Therefore, $n^* \geq \phi$ with strict inequality if $\mu > 1$.

Next, imposing the steady-state condition in 6:

$$\frac{1-n}{n} n^{\alpha} = (1-\phi)\left(\frac{n^{\alpha}}{n} + \frac{\mu - 1}{\mu}\frac{n^{*\alpha}}{n^*}\right)$$

Therefore, $n \leq \phi$ (since $\mu \geq 1$) with a strict inequality if $\mu > 1$. This establishes the first claim.

For the second claim, the ratio of wage (service producer to goods producer) in the home country is

$$\begin{aligned}\frac{w_t^2}{w_t^1} &= \frac{p_t^c}{p_t^k}\frac{(1-n)^{\alpha}}{n^{\alpha}} \\ &= \frac{1-n}{n} \\ &\geq \frac{1-\phi}{\phi},\end{aligned}$$

with equality if and only of $\mu = 1$. A similar argument establishes $\frac{w_t^{2*}}{w_t^{1*}} \leq \frac{1-\phi}{\phi}$. This establishes the second claim.

To establish the last claim, the equilibrium condition gives

$$\frac{\text{EXP}_t}{\text{GDP}_t} = \frac{p_{t-1}^k}{p_t^k}\frac{\mu n_{t-1}^{\alpha-1} + (\mu-1)\, n_{t-1}^{*\alpha-1}}{n_t^{\alpha-1}}$$

In steady state:

$$
\begin{aligned}
\frac{\mathrm{EXP}_t}{\mathrm{GDP}_t} &= \frac{1}{\mu}\frac{\mu n^{\alpha-1} + (\mu-1)\, n^{*\alpha-1}}{n^{\alpha-1}} \\
&= 1 + \frac{\mu-1}{\mu}\frac{n^{*\alpha}}{n^{\alpha}}\frac{n}{n^*} \\
&\geq 1
\end{aligned}
$$

with equality if and only if $\mu = 1$. A similar argument establishes $\mathrm{EXP}^*/\mathrm{GDP}^* \leq 1$.

A.3 Proof of Lemma 2

Let $K_{jt} = \sum_{i=1,2,3} k^i_{jt}$ for $j = H, F$ and $K^*_{Ft} = \sum_{i=1,2,3} k^{i*}_{Ft}$ be aggregate consumption of goods in the domestic and foreign economies, respectively. The allocation in the foreign economy must satisfy the following conditions.

$$
\begin{aligned}
p^{c*}_t (1-n^*_t)^{\alpha} &= (1-\phi)\left(p^{k*}_{t-1} n^{*\alpha}_{t-1} + p^{c*}_{t-1}\left(1-n^*_{t-1}\right)^{\alpha}\right) \\
p^{k*}_t K^*_{Ft} &= \phi\left(p^{k*}_{t-1} n^{*\alpha}_{t-1} + p^{c*}_{t-1}\left(1-n^*_{t-1}\right)^{\alpha}\right)
\end{aligned}
$$

From the optimal sectoral choice:

$$
\frac{p^{c*}_t}{p^{k*}_t} = \frac{n^{*\alpha}_t}{(1-n^*_t)^{\alpha}}\frac{(1-n^*_t)}{n^*_t}
$$

Using this:

$$
\begin{aligned}
n^{*\alpha-1}_t - n^{*\alpha}_t &= (1-\phi)\frac{p^{k*}_{t-1}}{p^{k*}_t} n^{*\alpha-1}_{t-1} \\
K^*_{Ft} &= \phi\frac{p^{k*}_{t-1}}{p^{k*}_t} n^{*\alpha-1}_{t-1}
\end{aligned}
$$

The first equation is (16), and the second is optimal demand for foreign-produced goods in the foreign economy. Allocation in the home economy must satisfy

$$
\begin{aligned}
p^c_t (1-n_t)^{\alpha} &= (1-\phi)\left(p^k_{t-1} n^{\alpha}_{t-1} + p^c_{t-1}(1-n_{t-1})^{\alpha} + (\mu-1) M_{t-1} + \theta p^{k*}_t K_{Ft}\right) \\
p^k_t K_{Ht} + (1+\theta) p^{k*}_t K_{Ft} &= \phi\left(p^k_{t-1} n^{\alpha}_{t-1} + p^c_{t-1}(1-n_{t-1})^{\alpha} + (\mu-1) M_{t-1} + \theta p^{k*}_t K_{Ft}\right)
\end{aligned}
$$

Replacing the equilibrium value of M_{t-1} and K_{Ht} and using the optimal sectoral choice condition, these simplify to

$$n_t^{\alpha-1} - n_t^{\alpha} = (1-\phi)\left(\mu \frac{p_{t-1}^k}{p_t^k} n_{t-1}^{\alpha-1} + (\mu-1)\frac{p_{t-1}^{k*}}{p_t^k} n_{t-1}^{*\alpha-1} + \theta \frac{p_t^{k*}}{p_t^k} K_{Ft}\right) \tag{45}$$

$$n_t^{\alpha} + (1+\theta)\frac{p_t^{k*}}{p_t^k} K_{Ft} = \phi\left(\mu \frac{p_{t-1}^k}{p_t^k} n_{t-1}^{\alpha-1} + (\mu-1)\frac{p_{t-1}^{k*}}{p_t^k} n_{t-1}^{*\alpha-1} + \theta \frac{p_t^{k*}}{p_t^k} K_{Ft}\right) \tag{46}$$

The equilibrium imports in the home country are

$$K_{Ft} = n_t^{*\alpha} - K_{Ft}^* = n_t^{*\alpha} - \phi \frac{p_{t-1}^{k*}}{p_t^{k*}} n_{t-1}^{*\alpha-1}$$

Substituting for K_{Ft} and $p_t^k = (1+\theta)p_t^{k*}$ gives equations (17) and (18).

A.4 Proof of Proposition 2

Part (i). Impose the steady-state condition $n_t^* = n^*$ in (16):

$$\frac{1-n^*}{n^*} n^{*\alpha} = (1-\phi)\frac{1}{\mu}\frac{n^{*\alpha}}{n^*},$$

which simplifies to $1 - n^* = (1-\phi)/\mu$, so $n^* = 1 - (1-\phi)/\mu$. This depends only on μ and ϕ; it is independent of θ.

Part (ii). Imposing steady state in (17) and substituting $p_t^k = (1+\theta)p_t^{k*}$, after substituting the expression for n^* and rearranging, the equilibrium condition for n reduces to

$$\phi \frac{n^{\alpha}}{n} - n^{\alpha} = (1-\phi)\left(\frac{\mu-1}{\mu}\right)\frac{n^{*\alpha}}{n^*}.$$

The right-hand side depends only on n^* (independent of θ by part (i)), μ, and ϕ. Hence n is independent of θ.

Part (iii). In steady state, foreign old-age goods demand is $k_F^* = (\phi/\mu)n^{*\alpha-1}$. Hence home imports are

$$K_F = n^{*\alpha} - \frac{\phi}{\mu} n^{*\alpha-1} = \frac{\mu-1}{\mu} n^{*\alpha-1},$$

which depends only on μ, ϕ, α, and n^*, all independent of θ. □

A.5 Proof of Proposition 3

Foreign allocation. The foreign equilibrium is unchanged relative to the benchmark. Hence $n^* = 1-(1-\phi)/\mu$ and $K_F = [(\mu-1)/\mu]n^{*\alpha-1}$, both independent of θ (cf. Proposition 2, parts (i) and (iii)).

Home labor allocation. From the steady-state home system (36)–(37), define $s \equiv (1+\theta)p^{k*}/p^k$. Equation (37) gives n as a strictly decreasing function of s:

$$n = \frac{(1-\eta)\phi}{(1-\phi)\eta\, s^{1-\rho} + (1-\eta)}.$$

Substituting into (36) gives a single equation in s whose solution is independent of θ because the right-hand side of (36) depends on θ only through s and K_F (both already pinned down). Hence n is independent of θ.

Trade-deficit ratio. The formula in the main text shows that the ratio equals (p^{k*}/p^k) times a quantity term that is fixed in steady state. It therefore falls with θ through the price ratio alone. □

A.6 Proof of Proposition 4

Part (i). The foreign problem is unchanged by κ. Hence $n^* = 1-(1-\phi)/\mu$ and $K_F = [(\mu-1)/\mu]n^{*\alpha-1}$, both independent of κ.

Part (ii). In steady state, the home equilibrium condition is

$$H(n,\kappa) \equiv (1+\kappa)\phi n^{\alpha-1} - (1+\phi\kappa)n^{\alpha} - (1-\phi)\frac{\mu-1}{\mu}n^{*\alpha-1} = 0.$$

By the implicit function theorem, $dn/d\kappa = -H_\kappa/H_n$. We have

$$H_\kappa = \phi n^{\alpha-1} - \phi n^{\alpha} = \phi n^{\alpha-1}(1-n) > 0,$$

and

$$H_n = (1+\kappa)\phi(\alpha-1)n^{\alpha-2} - (1+\phi\kappa)\alpha n^{\alpha-1} = n^{\alpha-2}\big[(1+\kappa)\phi(\alpha-1) - (1+\phi\kappa)\alpha n\big].$$

For $n \in (0,1)$ and $\alpha \in (0,1)$ both terms in brackets are negative, so $H_n < 0$. Therefore $dn/d\kappa = -H_\kappa/H_n > 0$. □

A.7 Proof of Proposition 5

Imposing steady state in the Case A system (Appendix B, equations (51)–(52)), the labor allocations satisfy $n^* = 1-(1-\phi)/\mu$ and $n = 1-(1-\phi)/[\mu(1-g)]$, exactly as in the benchmark but with endogenous μ. The money-growth rate is determined by the government budget constraint (57):

$$\big(\mu(1-g)-1\big)n^{\alpha-1} + \frac{\mu-1}{\mu}n^{*\alpha-1} = 0.$$

Substituting the expressions for n and n^*, define

$$\tilde{F}(\mu) \equiv \big((1-g)\mu-1\big)\left(\mu - \frac{1-\phi}{1-g}\right)^{\alpha-1} + (\mu-1+\phi)^{\alpha-1}(\mu-1).$$

The function $\tilde{F}$ is strictly increasing on $\left(\frac{1-\phi}{1-g}, \frac{1}{1-g}\right)$, with $\tilde{F} \to -\infty$ at the left endpoint and $\tilde{F} > 0$ at $\mu = \frac{1}{1-g}$. By the intermediate value theorem there is a unique $\hat{\mu} \in \left(\frac{1-\phi}{1-g}, \frac{1}{1-g}\right)$. One further verifies $\hat{\mu} > 1$. Since θ does not appear in $\tilde{F}$, $\hat{\mu}$, n, and n^* are all independent of θ. □

A.8 Proof of Proposition 6

Setup. Define the government budget constraint (66) as $G(\mu,\theta) = 0$, where

$$G(\mu,\theta) \equiv \big(\mu(1-g)-1\big)n(\mu,\theta)^{\alpha-1} + \frac{\mu-1}{1+\theta}n^*(\mu)^{\alpha-1},$$

with $n^*(\mu) = 1-(1-\phi)/\mu$ and $n(\mu,\theta)$ from (67).

Part (ii): $\mu'(\theta) > 0$. By the implicit function theorem, $\mu'(\theta) = -G_\theta/G_\mu$.

$G_\theta < 0$: The only direct θ-dependence in G is the factor $1/(1+\theta)$ in the second term, which is strictly decreasing in θ, making $G_\theta < 0$.

$G_\mu > 0$: Differentiating G with respect to μ, the first term contributes $(1-g)n^{\alpha-1} + (\alpha-1)(\mu(1-g)-1)n^{\alpha-2}\partial n/\partial\mu$ and the second term contributes $n^{*\alpha-1}/(1+\theta) + (\alpha-1)[(\mu-1)/(1+\theta)]n^{*\alpha-2}\partial n^*/\partial\mu$. Substituting $\partial n^*/\partial\mu = (1-\phi)/\mu^2 > 0$ and $\partial n/\partial\mu > 0$ (from (67)), all contributing terms to G_μ are positive, so $G_\mu > 0$.

Therefore $\mu'(\theta) = -G_\theta/G_\mu > 0$.

Part (i). Since $n^*(\theta) = 1-(1-\phi)/\mu(\theta)$ and $\mu'(\theta) > 0$, n^* is strictly increasing in θ.

Part (iii). Differentiating (67) with respect to θ:

$$\frac{dn}{d\theta} = \frac{1-\phi}{(1-g)\mu}\left[\frac{(1+\theta)\mu'(\theta)}{\mu} - (1-\mu(1-g))\right].$$

The sign is ambiguous because the first bracketed term is positive (raising n) while the second can be positive or negative.

Part (iv). As $\theta \to \infty$, examine (68): the factor $(\mu - 1)/(1 + \theta) \to 0$, so $G(\mu, \theta) = 0$ requires $\mu(1-g)-1 \to 0$, i.e. $\mu \to 1/(1-g)$. Substituting into (67): $n \to 1-(1-\phi)/[(1-g)\cdot 1/(1-g)] = 1 - (1 - \phi) = \phi$.

Part (v). For small g, evaluate $dn/d\theta$ at $\theta = 0$. At $\theta = 0$, $\mu(0) \equiv \mu_0 \in (1, 1/(1 - g))$ is close to 1 when g is small, and one can show that $\mu'(0) \cdot (1/\mu_0) < 1 - \mu_0(1 - g)$ for small g (verified numerically, e.g. $\phi = 0.5$, $\alpha = 0.5$, $g = 0.05$). Hence $dn/d\theta\big|_{\theta=0} < 0$, while $n(\infty) = \phi > n(0)$ (since $n(0) > \phi$ in the open economy with $\mu > 1$). By continuity, $n(\theta)$ must be non-monotone. □

B Alternative Fiscal Policies

B.1 Closed Economy

Consider the same economy as in the main text. The domestic government has exogenously given expenditures and consumes a constant fraction g of the output of goods and services. This expenditure is financed by issuing nominal asset M_t. Therefore, the domestic government's budget constraint is

$$M_t - M_{t-1} = g\left(p_t^c\left(1 - n_t\right)^\alpha + p_t^k n_t^\alpha\right) \tag{47}$$

Here, g is the exogenous policy choice, and the path of M_t that finances the expenditure is an endogenous equilibrium object.

Definition. *A closed economy competitive equilibrium is a sequence of worker consumption allocations and money holdings* $\{c_t^i, k_t^i, m_t^i\}_{t=0,i\in\{1,2,3\}}^\infty$, *the fraction of type 3 households in the goods sector* $\{n_t\}_{t=0}^\infty$, *prices* $\{p_t^c, p_t^k\}_{t=0}^\infty$, *wages* $\{w_t^i\}_{t=0,i\in\{1,2,3\}}^\infty$, *and government policy* $\{M_t, g\}_{t=0}^\infty$, *such that*

1. Given prices, wages and government policy, allocation $\{c_t^i, k_t^i\}_{t=0}^\infty$ solves the problem of individual worker i'

$$\max u\left(c_{t+1}^i, k_{t+1}^i\right)$$

s.t.

$$m_t^i \le w_t^i$$
$$p_{t+1}^c c_{t+1}^i + p_{t+1}^k k_{t+1}^i \le m_t^i$$

2. The wages are determined according to equations (1), (2) and (3).

3. The government budget constraint (47) holds.

4. Markets clear

$$\sum_{i=1,2,3} c_t^i = (1-g)(1-n_t)^\alpha$$
$$\sum_{i=1,2,3} k_t^i = (1-g)\, n_t^\alpha$$
$$\sum_{i=1,2,3} m_t^i = M_t$$

Optimal demand by old households is given by

$$p_{t+1}^k (1-g)\, n_{t+1}^\alpha = \phi M_t \tag{48}$$
$$p_{t+1}^c (1-g)(1-n_{t+1})^\alpha = (1-\phi) M_t \tag{49}$$

This implies

$$\frac{p_{t+1}^k}{p_{t+1}^c} \frac{n_{t+1}^\alpha}{(1-n_{t+1})^\alpha} = \frac{\phi}{1-\phi}.$$

Optimal sector choice by young mobile households implies

$$\frac{p_{t+1}^k}{p_{t+1}^c} = \frac{(1-n_{t+1})^{\alpha-1}}{n_{t+1}^{\alpha-1}}.$$

Combining these equations yields

$$n_{t+1} = \phi.$$

Total income of young households in period t equals total demand for assets:

$$M_t = p_t^k n_t^\alpha + p_t^c (1-n_t)^\alpha = p_t^k n_t^{\alpha-1},$$

where the second equality follows from optimal sector choice. Total expenditures in period $t+1$ equal the asset position of old households. Aggregate private consumption of goods and services in period $t+1$ is $(1-g)\, n_t^\alpha$ and $(1-g)(1-n_t)^\alpha$, respectively. Therefore,

$$M_t = p_{t+1}^k (1-g)\, n_{t+1}^\alpha + p_{t+1}^c (1-g)(1-n_{t+1})^\alpha = p_{t+1}^k n_{t+1}^{\alpha-1}$$

Combining these equations and replacing $n_t = \phi$ gives

$$\frac{p_{t+1}^k}{p_t^k} = \frac{1}{1-g}.$$

and therefore

$$\mu \equiv \frac{M_{t+1}}{M_t} = \frac{1}{1-g}.$$

In the closed economy, money grows at a constant rate $\mu = \frac{1}{1-g}$, resulting in a constant inflation rate $\frac{p^c_{t+1}}{p^c_t} = \frac{p^k_{t+1}}{p^k_t} = \frac{1}{1-g}$.

B.2 Trade: Tariff is used to finance deficit

We now introduce a foreign country. As in the main text, only the domestic economy can issue money. The foreign economy must export goods to the domestic economy to acquire money; otherwise no production takes place in the foreign economy.

The home government levies a tariff at rate θ on imported goods. Revenue from tariffs is used to finance expenditures.

Definition. *A tariff distorted competitive equilibrium is a sequence of*

domestic and foreign allocations and money holdings
$\left\{\left(c^i_t, k^i_{Ht}, k^i_{Ft}, m^i_t\right), \left(c^{i*}_t, k^{i*}_{Ft}, m^{i*}_t\right)\right\}^\infty_{t=0,\ i\in\{1,2,3\}}$,
mobile-labor allocations $\{n_t, n^*_t\}^\infty_{t=0}$,
prices $\left\{p^c_t, p^{c*}_t, p^k_t, p^{k*}_t\right\}^\infty_{t=0}$,
wages $\left\{w^i_t, w^{i*}_t\right\}^\infty_{t=0,\ i\in\{1,2,3\}}$,
and government policy $\{\theta, M_t, g\}^\infty_{t=0}$,

such that

1. Given prices, wages and government policy, allocation $\{c^i_t, k^i_t, m^i_t\}^\infty_{t=0}$ solves the problem of domestic individual worker i'

$$\max u\left(c^i_{t+1}, k^i_{Ht+1} + k^i_{Ft+1}\right)$$

s.t.

$$m^i_t \leq w^i_t$$
$$p^c_{t+1}c^i_{t+1} + p^k_{t+1}k^i_{Ht+1} + (1+\theta)\, p^{k*}_{t+1}k^i_{Ft+1} \leq m^i_t$$

2. Given prices, wages, and government policy, allocation $\{c^{i*}_t, k^{i*}_t, m^{i*}_t\}^\infty_{t=0}$ solves the problem of the foreign individual worker i' (note: the difference is that they do not receive the

transfer)

$$\max u\left(c^{i*}_{t+1}, k^{i*}_{Ft+1}\right)$$

s.t.

$$\begin{aligned} m^{i*}_t &\leq w^{i*}_t \\ p^{c*}_{t+1} c^{i*}_{t+1} + p^{k*}_{t+1} k^{i*}_{Ft+1} &\leq m^{i*}_t \end{aligned}$$

3. Domestic wages are determined according to equations (1), (2) and (3) (and similarly for foreign wages).

4. The government budget constraint holds

$$M_t - M_{t-1} + \theta p^{k*}_t \left(\sum_{i=1,2,3} k^i_{Ft}\right) = g\left(p^c_t \left(1-n_t\right)^\alpha + p^k_t n^\alpha_t\right) \tag{50}$$

5. Allocation is feasible

$$\begin{aligned} \sum_{i=1,2,3} c^i_t &= \left(1-g\right)\left(1-n_t\right)^\alpha \\ \sum_{i=1,2,3} c^{i*}_t &= \left(1-n^*_t\right)^\alpha \\ \sum_{i=1,2,3} k^i_{Ht} &= \left(1-g\right) n^\alpha_t \\ \sum_{i=1,2,3} k^i_{Ft} + \sum_{i=1,2,3} k^{i*}_{Ft} &= n^{*\alpha}_t \\ \sum_{i=1,2,3} m^i_t + \sum_{i=1,2,3} m^{i*}_t &= M_t \end{aligned}$$

The law of one price must hold in equilibrium: $p^k_t = \left(1+\theta\right) p^{k*}_t$. Imposing market clearing and households' optimal choices of service consumption:

$$p^{c*}_{t+1}\left(1-n^*_{t+1}\right)^\alpha = \left(1-\phi\right)\left(p^{k*}_t \left(n^*_t\right)^\alpha + p^{c*}_t \left(1-n^*_t\right)^\alpha\right) \tag{51}$$

$$\left(1-g\right) p^c_{t+1}\left(1-n_{t+1}\right)^\alpha = \left(1-\phi\right)\left(p^k_t n^\alpha_t + p^c_t \left(1-n_t\right)^\alpha\right) \tag{52}$$

Imposing steady state and using optimal sector choice equations (28) and (32):

$$\mu(1-n^*)^\alpha = (1-\phi)(1-n^*)^{\alpha-1}$$
$$(1-g)\,\mu(1-n)^\alpha = (1-\phi)(1-n)^{\alpha-1}$$

Therefore,

$$n^* = 1 - \frac{1-\phi}{\mu}$$
$$n = 1 - \frac{1-\phi}{\mu(1-g)}$$

The labor allocation in the goods sector is always higher in the foreign economy.

To solve for the unknown growth rate μ, the aggregate demand for goods in the foreign economy is

$$p^{k*}_{t+1}k^*_{Ft+1} = \phi\left(p^{k*}_t (n^*_t)^\alpha + p^{c*}_t (1-n^*_t)^\alpha\right) \tag{53}$$

which simplifies to

$$k^*_{Ft+1} = \frac{\phi}{\mu}(n^*_t)^{\alpha-1}.$$

Home imports in steady state are

$$(n^*)^\alpha - k^*_F = \left(\frac{\mu-1}{\mu}\right)(n^*)^{\alpha-1} \tag{54}$$

The total demand for assets by home and foreign young households must equal the supply:

$$\begin{aligned} M_t &= p^k_t n^\alpha_t + p^c_t(1-n_t)^\alpha + p^{k*}_t(n^*_t)^\alpha + p^{c*}_t(1-n^*_t)^\alpha \\ &= p^k_t\left(n^{\alpha-1}_t + \frac{(n^*_t)^{\alpha-1}}{1+\theta}\right) \end{aligned} \tag{55}$$

Substituting into the government budget constraint:

$$p^k_t\left(n^{\alpha-1}_t + \frac{(n^*_t)^{\alpha-1}}{1+\theta}\right) - p^k_{t-1}\left(n^{\alpha-1}_{t-1} + \frac{(n^*_{t-1})^{\alpha-1}}{1+\theta}\right) = p^k_t\left(g n^{\alpha-1}_t + \frac{\theta}{1+\theta}\left((n^*_t)^\alpha - \frac{\phi}{\mu}(n^*_{t-1})^{\alpha-1}\right)\right) \tag{56}$$

Imposing steady state and rearranging:

$$\left(1-g-\frac{1}{\mu}\right)n^{\alpha-1}+\left(\frac{\mu-1}{\mu}\right)(n^{*})^{\alpha-1}=0 \tag{57}$$

where $n^{*}=1-\frac{1-\phi}{\mu}$ and $n=1-\frac{1-\phi}{\mu(1-g)}$.

The tariff has no long-term impact on allocation.

Proposition 7. *In equilibrium, there is a unique inflation rate* μ *such that* $1<\mu<\frac{1}{1-g}$.

Proof. Define

$$F(\mu)=((1-g)\mu-1)\left(\mu-\frac{1-\phi}{1-g}\right)^{\alpha-1}+(\mu-1+\phi)^{\alpha-1}(\mu-1)$$

Step 1. The equation above has a unique solution in the interval $\left(\frac{1-\phi}{1-g},\frac{1}{1-g}\right)$.

$F(\cdot)$ is monotonically increasing on the interval $\left(\frac{1-\phi}{1-g},\frac{1}{1-g}\right)$:

$$\begin{aligned}
F'(\mu)&=(1-g)\left(\mu-\frac{1-\phi}{1-g}\right)^{\alpha-1}+(\alpha-1)((1-g)\mu-1)\left(\mu-\frac{1-\phi}{1-g}\right)^{\alpha-2}\\
&+(\mu-1+\phi)^{\alpha-1}+(\alpha-1)(\mu-1+\phi)^{\alpha-2}(\mu-1)\\
&=\left(\mu-\frac{1-\phi}{1-g}\right)^{\alpha-2}\left((1-g)\left(\mu-\frac{1-\phi}{1-g}\right)+(\alpha-1)((1-g)\mu-1)\right)\\
&+(\mu-1+\phi)((\mu-1+\phi)+(\alpha-1)(\mu-1))\\
&=\left(\mu-\frac{1-\phi}{1-g}\right)^{\alpha-2}((1-g)\mu-(1-\phi)+(\alpha-1)((1-g)\mu-1))\\
&+(\mu-1+\phi)^{\alpha-2}(\mu-(1-\phi)+(\alpha-1)(\mu-1))\\
&>0
\end{aligned}$$

where the last inequality holds because both terms are positive for $\mu\in\left(\frac{1-\phi}{1-g},\frac{1}{1-g}\right)$.

Evaluating F on the boundary:

$$F\left(\frac{1-\phi}{1-g}\right)=-\phi\left(\frac{1-\phi}{1-g}-\frac{1-\phi}{1-g}\right)^{\alpha-1}+\left(\frac{1-\phi}{1-g}-1+\phi\right)^{\alpha-1}\left(\frac{1-\phi}{1-g}-1\right)=-\infty$$

and

$$F\left(\frac{1}{1-g}\right)=\left(\frac{1}{1-g}-1+\phi\right)^{\alpha-1}\left(\frac{1}{1-g}-1\right)>0$$

By the intermediate value theorem, there is a unique $\hat{\mu}$ such that $F(\hat{\mu})=0$.

Step 2. $\hat{\mu} > 1$.

In the equation

$$((1-g)\mu - 1)\left(\mu - \frac{1-\phi}{1-g}\right)^{\alpha-1} + \frac{1}{1+\theta}(\mu - 1 + \phi)^{\alpha-1}(\mu - 1) = 0$$

we know $\hat{\mu} < \frac{1}{1-g}$, so $((1-g)\hat{\mu} - 1)\left(\hat{\mu} - \frac{1-\phi}{1-g}\right)^{\alpha-1} < 0$. If $\hat{\mu} \leq 1$, then $F(\hat{\mu}) < 0$, contradicting the definition of $\hat{\mu}$. Therefore,

$$1 < \hat{\mu} < \frac{1}{1-g}.$$

□

B.3 Trade: Tariff revenue is rebated to the old

Consider now an alternative fiscal policy in which tariff revenue is used to finance a lump-sum transfer to the old. The problem of households in the home economy is

$$\max u\left(c_{t+1}^i, k_{Ht+1}^i + k_{Ft+1}^i\right)$$

s.t.

$$m_t^i \leq w_t^i$$

$$p_{t+1}^c c_{t+1}^i + p_{t+1}^k k_{Ht+1}^i + (1+\theta) p_{t+1}^{k*} k_{Ft+1}^i \leq m_t^i + \tau_t^i$$

and the government budget constraints are

$$M_t - M_{t-1} = g\left(p_t^c (1-n_t)^\alpha + p_t^k n_t^\alpha\right) \tag{58}$$

$$\sum_{i=1,2,3} \tau_t^i = \theta p_t^{k*}\left(\sum_{i=1,2,3} k_{Ft}^i\right) \tag{59}$$

The equilibrium can be defined in a similar fashion. The problem of foreign households is the same as before. Aggregate demand for goods by foreigners is

$$k_{Ft+1}^* = \frac{\phi}{\mu}(n_t^*)^{\alpha-1}.$$

Home imports must therefore be

$$\left(n_{t+1}^{*}\right)^{\alpha}-k_{Ft+1}^{*}=\left(n_{t+1}^{*}\right)^{\alpha}-\frac{\phi}{\mu}\left(n_{t}^{*}\right)^{\alpha-1}.$$

The following equations characterize the path of equilibrium allocations:

$$p_{t+1}^{c*}\left(1-n_{t+1}^{*}\right)^{\alpha}=\left(1-\phi\right)\left(p_{t}^{k*}\left(n_{t}^{*}\right)^{\alpha}+p_{t}^{c*}\left(1-n_{t}^{*}\right)^{\alpha}\right) \tag{60}$$

$$\left(1-g\right)p_{t+1}^{c}\left(1-n_{t+1}\right)^{\alpha}=\left(1-\phi\right)\Bigg(p_{t}^{k}n_{t}^{\alpha}+p_{t}^{c}\left(1-n_{t}\right)^{\alpha}+\theta p_{t+1}^{k*}\left(\left(n_{t+1}^{*}\right)^{\alpha}-\frac{\phi}{\mu}\left(n_{t}^{*}\right)^{\alpha-1}\right)\Bigg) \tag{61}$$

$$\left(p_{t+1}^{k}k_{t+1}+p_{t+1}^{k*}k_{t+1}^{*}\right)=\phi\Bigg(p_{t}^{k}n_{t}^{\alpha}+p_{t}^{c}\left(1-n_{t}\right)^{\alpha}+\theta p_{t+1}^{k*}\left(\left(n_{t+1}^{*}\right)^{\alpha}-\frac{\phi}{\mu}\left(n_{t}^{*}\right)^{\alpha-1}\right)+p_{t}^{k*}\left(n_{t}^{*}\right)^{\alpha}+p_{t}^{c*}(1-n_{t}^{*})^{\alpha}\Bigg) \tag{62}$$

and the government budget constraint

$$M_{t+1}-M_{t}=g\left(p_{t+1}^{c}\left(1-n_{t+1}\right)^{\alpha}+p_{t+1}^{k}n_{t+1}^{\alpha}\right)$$

which can be written as (using (32))

$$p_{t+1}^{k}\left(n_{t+1}^{\alpha-1}+\frac{\left(n_{t+1}^{*}\right)^{\alpha-1}}{1+\theta}\right)-p_{t}^{k}\left(n_{t}^{\alpha-1}+\frac{\left(n_{t}^{*}\right)^{\alpha-1}}{1+\theta}\right)=gp_{t+1}^{k}n_{t+1}^{\alpha-1}$$

In steady state:

$$\mu\left(1-n^{*}\right)^{\alpha}=\left(1-\phi\right)\left(1-n^{*}\right)^{\alpha-1} \tag{63}$$

$$\left(1-g\right)n^{\alpha-1}\left(1-n\right)=\left(1-\phi\right)\left(\frac{1}{\mu}n^{\alpha-1}+\frac{\theta}{1+\theta}\frac{\mu-1}{\mu}\left(n^{*}\right)^{\alpha-1}\right) \tag{64}$$

$$\left(k+\frac{k^{*}}{1+\theta}\right)=\phi p_{t}^{k}\left(\frac{1}{\mu}n^{\alpha-1}+\frac{\theta}{1+\theta}\frac{\mu-1}{\mu}\left(n^{*}\right)^{\alpha-1}+\frac{\left(n_{t}^{*}\right)^{\alpha-1}}{1+\theta}\right) \tag{65}$$

and government budget constraint in steady state:

$$\left(1-g-\frac{1}{\mu}\right)n^{\alpha-1}+\left(\frac{\mu-1}{\mu}\right)\frac{\left(n^{*}\right)^{\alpha-1}}{1+\theta}=0. \tag{66}$$

The foreign labor allocation is

$$n^* = 1 - \frac{1-\phi}{\mu}.$$

Combining government budget constraint (66) and (64) in the home economy:

$$n = 1 - \left(\frac{1-\phi}{1-g}\right)\left(\frac{1+\theta}{\mu} - \theta(1-g)\right) \tag{67}$$

Substituting back into the government budget constraint:

$$(\mu(1-g)-1)\left(1 - \left(\frac{1-\phi}{1-g}\right)(1+\theta-\mu\theta(1-g))\right)^{\alpha-1} + \left(\frac{\mu-1}{1+\theta}\right)(\mu-1+\phi)^{\alpha-1} = 0. \tag{68}$$

Using the same argument as above, this equation has a unique solution in $\left(1, \frac{1}{1-g}\right)$. One can also show that $\mu'(\theta) > 0$. The allocation of labor at home is not monotone in the tariff.

C Symmetric Armington: Two-Way Trade

This appendix develops the symmetric Armington extension summarized in Section 4. Both countries have CES demand over home and foreign tradable varieties. Let $q \equiv p^k/p^{k*}$ denote the relative producer price of the home variety. Home households have Armington elasticity $\rho > 1$ and home-variety weight $\eta \in (0,1)$; foreign households have elasticity $\gamma > 1$ and foreign-variety weight $1-\eta^*$ for $\eta^* \in (0,1)$. The home country levies an ad valorem tariff $\theta \geq 0$ on the foreign variety, so the tariff-inclusive home import price is $(1+\theta)p^{k*}$.

Lemma 3 (Armington import ratios)**.** *In an interior allocation, Armington demand implies*

$$\frac{k_F}{k_H} = \frac{1-\eta}{\eta}\left(\frac{1+\theta}{q}\right)^{-\rho}, \tag{69}$$

$$\frac{k_H^*}{k_F^*} = \frac{\eta^*}{1-\eta^*}q^{-\gamma}. \tag{70}$$

Proof. Standard CES cost minimization for the tradables aggregator in each country, accounting for the tariff-inclusive price $(1+\theta)p^{k*}$ facing home consumers. □

Lemma 4 (Gross flows given (q,θ))**.** *Let $a(q,\theta) \equiv \frac{1-\eta}{\eta}(1+\theta)^{-\rho}q^{\rho}$ and $b(q) \equiv \frac{\eta^*}{1-\eta^*}q^{-\gamma}$. Market* *clearing for each variety yields*

$$k_F(q,\theta) = \frac{a(q,\theta)\big(n^{\alpha} - b(q)n^{*\alpha}\big)}{1 - a(q,\theta)b(q)}, \tag{71}$$

$$k_H^*(q,\theta) = b(q)\big(n^{*\alpha} - k_F(q,\theta)\big). \tag{72}$$

Holding q fixed, k_F is strictly decreasing in θ. Under $ab < 1$, k_H^* is strictly decreasing in q.

Proof. Substitute (70) into the market-clearing condition for the home variety and solve the resulting 2×2 system; monotonicity follows from $\partial a/\partial\theta < 0$ and $\partial b/\partial q < 0$. □

In steady state, the reserve-currency mechanism pins down $n^* = 1-(1-\phi)/\mu$ and therefore the net real resource transfer $T = (1 - 1/\mu)n^{*\alpha-1}$. The balance-of-payments identity requires

$$k_F - q\,k_H^* = T, \tag{73}$$

which is independent of θ. Define $\mathcal{F}(q,\theta) \equiv k_F(q,\theta) - q\,k_H^*(q,\theta) - T$; an equilibrium relative price $q(\theta)$ solves $\mathcal{F}(q,\theta) = 0$.

Proposition 8 (Terms-of-trade appreciation). *If $\mathcal{F}_q > 0$ at an equilibrium (sufficient condition: $ab < 1$ and $\rho > \gamma$), then $dq/d\theta > 0$.*

Proof. By the IFT, $dq/d\theta = -\mathcal{F}_\theta/\mathcal{F}_q$. From Lemma 4, $\mathcal{F}_\theta = \partial k_F/\partial\theta < 0$ at fixed q. When $\rho > \gamma$ and $ab < 1$, the home substitution channel dominates the foreign channel, giving $\mathcal{F}_q > 0$, and hence $dq/d\theta > 0$. □

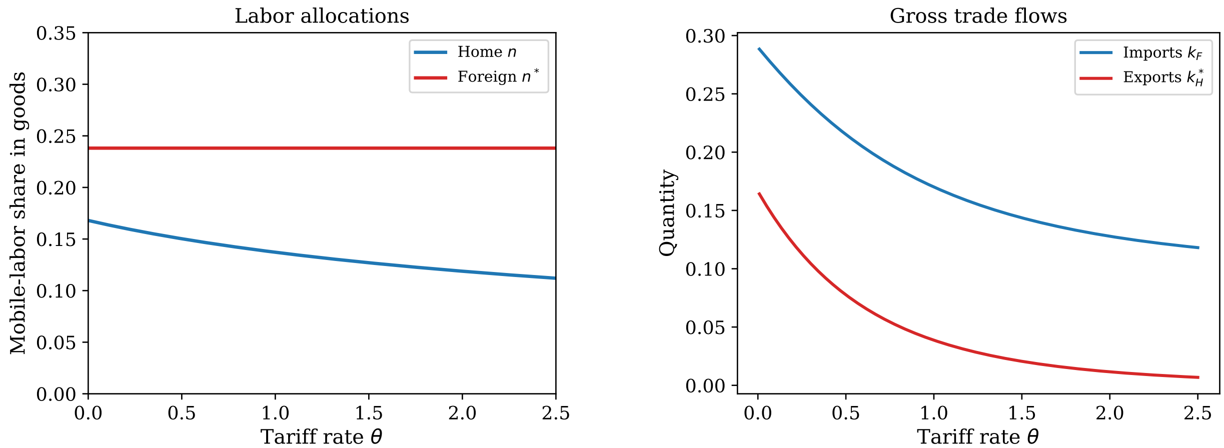
Proposition 9. *If the conditions of Proposition 8 hold and the incomplete-offset condition*

$$\frac{d}{d\theta}\ln\left(\frac{1+\theta}{q}\right) > 0,$$

both $dk_F/d\theta < 0$ and $dk_H^*/d\theta < 0$.

Proof. The effective relative import price $(1+\theta)/q(\theta)$ rises by assumption, reducing home import demand ($\rho > 1$). The home relative producer price q rises, reducing foreign demand for the home variety ($\gamma > 1$). □

Proposition 10 (Home deindustrialization under tariffs). *In the symmetric Armington environment, under the incomplete-offset condition, $dn/d\theta < 0$: a tariff further reduces the home tradables labor share.*



(a) Labor allocations.

(b) Gross trade flows.

Figure 6: Armington comparative statics ($\phi = 0.2$, $\alpha = 0.5$, $\mu = 1.05$, $\rho = \gamma = 4$, $\eta = \eta^* = 0.6$). As the import tariff θ increases, the home goods-sector labor share n declines (further deindustrialization, left panel), while the foreign goods-sector labor share n^* remains unchanged. Both gross trade flows—imports k_F and exports k_H^*—fall monotonically with θ (right panel).

Proof. The tariff raises the home CES price index for tradables, shifting home expenditure toward services and lowering the equilibrium relative price p^k/p^c. Since the home labor supply schedule is $p^k/p^c = [(1-n)/n]^{\alpha-1}$ (upward-sloping in n), a lower p^k/p^c requires a lower n. □

Figure 6 reports a numerical comparative statics exercise. As the tariff increases: (i) the foreign goods-sector labor share is essentially unchanged; (ii) the home goods-sector labor share declines (further deindustrialization); and (iii) both gross trade flows fall.