Tariff Policy with an Exorbitant Privilege*

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March 27, 2026

Abstract

We study tariffs in a reserve-currency economy where foreign demand for the dominant transaction asset finances persistent net exports to the issuer and shrinks its tradables sector over time. In this environment, a unilateral import tariff does not reindustrialize the economy: it leaves long-run tradables employment, output, and import quantities unchanged. Its main effects are on relative prices, fiscal revenue, and the measured trade-deficit-to-GDP ratio, which falls through a valuation effect rather than real trade rebalancing. This neutrality extends to a multi-country setting with a competing settlement currency and endogenous portfolio choice.

JEL Classification: F13, F32, F41, E42.
Keywords: reserve currency; exorbitant privilege; tariffs; trade deficit.

1 Introduction

The United States runs a persistent trade deficit, and tariffs are frequently proposed as the remedy. Whether tariffs can reduce that deficit, however, depends on what the deficit represents. We argue that, in a reserve-currency economy, a persistent trade deficit is the equilibrium counterpart of foreign demand for domestic transaction assets. If that is the relevant margin, then the question is not whether tariffs can alter import prices, but whether they can overturn the foreign demand for the dominant currency. In our framework, the answer is no.

We study this in what we call an "exorbitant privilege" environment. A country whose liabilities are valued as safe and liquid stores of value can finance a real resource transfer from the rest

*We thank participants at the Third Jackson Hole International Finance and Macroeconomics Conference for useful comments and suggestions. Remaining errors are ours.

of the world. Foreigners accumulate the dominant transaction asset by shipping tradable goods in exchange. That transfer has real allocation consequences: it changes relative prices between tradables and nontradables and, through those prices, shifts labor away from the home tradables sector. In this sense, reserve-currency status can generate a long-run form of deindustrialization.

This perspective differs from a prominent view in current policy debates, where the trade deficit is treated as a price distortion and tariffs as the natural corrective instrument. Recent proposals associated with Miran Miran (2024) and the so-called "Mar-a-Lago Accord" are examples of this broader argument. In that view, broad import tariffs redirect spending toward domestic producers, shrink the import bill, and perhaps force some of the burden onto foreign exporters. That logic is intuitive in partial equilibrium. The question we ask is whether it survives in a general equilibrium in which the trade deficit is pinned down by foreign asset demand rather than by a mispriced import wedge.

We study that question in a stylized two-country dynamic general equilibrium model. The two countries are otherwise identical, except that the home country issues the only internationally accepted transaction asset. Households consume a tradable good and a nontradable service, and production is Ricardo–Viner: each sector has a sector-specific factor and a mobile factor that can move across sectors. The key distinction is between the goods-price margin, which tariffs affect, and the asset-demand margin, which determines the real transfer associated with reserve-currency status. We use a labor-supply/labor-demand representation to make the sectoral implications of that distinction transparent.

The central result is that a unilateral import tariff does not reverse the long-run contraction of the home tradables sector. On the foreign side, the quantity of tradables exported to the home country is governed by demand for the home transaction asset, not by the tariff wedge. This pins down both the foreign labor allocation and the long-run quantity of home imports independently of the tariff.

On the home side, the tariff raises the consumer price of imported goods, but if tariff revenue is rebated lump-sum to home consumers, their purchasing power rises one-for-one. The relative return to producing tradables at home is therefore unchanged, so long-run tradables employment and output are unchanged as well.

What the tariff does change is valuation. A higher tariff drives a wedge between the price paid by home consumers and the price received by foreign producers. Because trade statistics value imports at world prices while domestic GDP is measured at domestic prices, this wedge mechanically lowers the measured trade-deficit-to-GDP ratio even when import quantities do not change. In this sense, the tariff is a valuation device rather than a real rebalancing device.

The tariff also redistributes income. Foreign exporters receive lower net-of-tariff prices, home consumers pay more for imported goods, and the home government collects the difference. The burden is therefore shared between domestic consumers and foreign producers. Tariffs are not irrelevant in this environment: they raise revenue and shift international income. But they do not eliminate the real trade deficit or reindustrialize the home economy, because those outcomes are governed by asset demand rather than by the border price of imports.

We also show that this neutrality result is not an artifact of the benchmark assumption that the home transaction asset is the only store of value. In a multi-country extension with differentiated goods, a competing foreign settlement instrument, and endogenous portfolio choice between currencies, the same logic survives. A goods tariff operates on the goods-price margin, while the trade deficit is pinned down by the asset-return margin that governs portfolio allocation. By contrast, a financial tariff on foreign reserve holdings operates directly on that asset-return margin and therefore does reduce the trade deficit and expand home tradables employment. The broader message is that, in a reserve-currency economy, the effectiveness of policy depends on which margin it targets.

1.1 Related literature

Our paper is related to three strands of literature.

The first studies the macroeconomic consequences of the dollar's international role. Gourinchas and Rey Gourinchas and Rey (2005, 2013) document that the United States functions as a world banker and that external adjustment occurs through valuation effects as well as trade flows. Caballero, Farhi, and Gourinchas Caballero et al. (2017, 2016) rationalize persistent imbalances as an equilibrium response to global safe-asset scarcity, while Krishnamurthy and Vissing-Jorgensen Krishnamurthy and Vissing-Jorgensen (2012) document the convenience yield on U.S. Treasuries. The closest theoretical predecessor is Farhi and Maggiori Farhi and Maggiori (2018), who study a reserve-currency issuer in the international monetary system. We share their core asymmetry, but focus on a two-sector environment in which reserve-currency demand translates into sectoral labor reallocation and long-run deindustrialization. Bernanke's Bernanke (2005) global saving glut hypothesis and Obstfeld and Rogoff Obstfeld and Rogoff (2005) provide related perspectives on persistent external imbalances. Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller Gopinath et al. (2020) highlight a complementary channel through dominant-currency pricing.

The second strand studies tariffs, terms of trade, and trade balances. The Lerner symmetry theorem Lerner (1936) limits what broad-based tariffs can achieve without accompanying shifts in saving or investment, and the optimal-tariff literature Metzler (1949); Bagwell and Staiger (1999, 2002) emphasizes terms-of-trade effects for large countries. Our model shows how those classic insights interact with reserve-currency asymmetry. Terms-of-trade movements remain important, but in our environment they primarily operate through valuation and revenue redistribution rather than through long-run quantity rebalancing. The Ricardo–Viner production structure Jones (1971) makes the sectoral consequences of that interaction transparent.

The third strand emphasizes valuation, incidence, and sectoral adjustment. Caliendo, Kortum, and Parro Caliendo et al. (2025) and Itskhoki and Mukhin Itskhoki and Mukhin (2025) stress valuation channels in interpreting tariff-induced movements in measured trade deficits. Amiti, Redding, and Weinstein Amiti et al. (2019) and Fajgelbaum, Goldberg, Kennedy, and Khandelwal Fajgelbaum et al. (2020) document substantial pass-through of tariffs into domestic prices and shared incidence between domestic consumers and foreign exporters. Autor, Dorn, and Hanson Autor et al. (2013) show that import competition induces persistent local labor-market adjustment away from tradables. Our contribution is to connect these ideas in a reserve-currency setting: the real quantity of net imports is pinned down by asset demand, tariffs move measured deficits through valuation, and the long-run contraction of home tradables employment reflects reserve-currency demand rather than a correctable import-price distortion.

In complementary and independent work, Bigio (2026) studies tariff policy in a reserve-currency economy with capital mobility and money-in-the-utility. Although his mechanism differs from ours, his analysis also points to the limited usefulness of goods tariffs relative to financial policy. Our paper differs by emphasizing sectoral reallocation, deindustrialization, and the distinction between goods-price and asset-return margins in an OLG environment with two production sectors.

Road map. Section 2 develops the model and characterizes the reserve-currency equilibrium. Section 3 studies tariff policy in the benchmark environment, including the neutrality result, the valuation channel, and tariff incidence. Section 4 presents extensions on imperfect substitution, industrial policy, alternative fiscal policy, and a multi-country setting with endogenous portfolio choice. Section 5 concludes. Proofs and additional extensions are collected in the appendix.

2 Model

2.1 Closed Economy

We begin with a closed economy. Agents live for two periods. The young work; the old consume. There are two production sectors—goods and services—and three types of workers. Type 1 is specific to goods, type 2 is specific to services, type 3 is mobile and can work in either. There is one unit of each type.

Let c_t denote services and let k_t denote goods. Let $0 \leq n_t \leq 1$ denote the measure of type

3 (mobile) labor employed in the goods sector. Goods are produced using input from the type 1 worker (fixed) and input from the type 3 worker according to the production function $k_t = n_t^{\alpha}$. Since input of type 1 is fixed, we suppress it as an argument and treat it as a factor in fixed supply. Services are produced using input from type 2 and type 3 workers according to $c_t = G(1 - n_t)$. Let p_t^c and p_t^k denote the money price of services and goods, respectively, and let w_t^3 be the wage of the mobile worker. Indifference of the mobile worker across sectors requires

$$w_t^3 = p_t^c G'(1 - n_t) = p_t^k F'(n_t)$$

For simplicity, assume $G = F$ and $F(n) = n^{\alpha}$, $0 < \alpha < 1$, so that $F'(n) = \alpha F(n)/n$ and $F(n) - nF'(n) = (1-\alpha) F(n)$.[1] The wages paid to each type are then

$$w_t^1 = p_t^k (1-\alpha) n_t^{\alpha} \quad (1)$$

$$w_t^2 = p_t^c (1-\alpha)(1-n_t)^{\alpha} \quad (2)$$

$$w_t^3 = p_t^k \alpha n_t^{\alpha-1} = p_t^c \alpha (1-n_t)^{\alpha-1} \quad (3)$$

Individuals have identical preferences over future goods and services,

$$U_t = u(c_{t+1}, k_{t+1}).$$

Young households do not value current consumption, so they save all their income. Type 1 and 2 households have a trivial labor supply choice. Type 3 households are indifferent in equilibrium across sectors and earn the same wage w_t^3 in either one.[2]

Goods and services are non-storable. The single asset is nominal government debt (fiat money). Let M_t denote the money supply at date t, where $M_t = \mu M_{t-1}$ with M_0 in the hands of the initial old. New money $\tau_t = (1 - 1/\mu) M_t$ is injected as a lump-sum transfer to the old and divided equally among types. Let m_t^i be young household i's money holdings.

Definition 1. *A closed economy competitive equilibrium is a sequence of consumption allocations and money holdings* $\{c_t^i, k_t^i, m_t^i\}_{t=0, i\in\{1,2,3\}}^{\infty}$*, a fraction of type 3 households in the goods sector* $\{n_t\}_{t=0}^{\infty}$*, prices* $\{p_t^c, p_t^k\}_{t=0}^{\infty}$*, wages* $\{w_t^i\}_{t=0, i\in\{1,2,3\}}^{\infty}$*, and a government policy* $\{M_t, \tau_t\}_{t=0}^{\infty}$*, such that*

1. *Given prices, wages and government policy, allocation* $\{c_t^i, k_t^i\}_{t=0}^{\infty}$ *solves the problem of individual worker* i*'s decision problem* $\max u\left(c_{t+1}^i, k_{t+1}^i\right)$ *subject to:*

$$m_t^i \leq w_t^i \text{ and } p_{t+1}^c c_{t+1}^i + p_{t+1}^k k_{t+1}^i \leq m_t^i + \frac{\tau_{t+1}}{3}$$

[1] Different types of labor are not substitutes for one another.

[2] Technically, the young value their time a little bit, to avoid having them supply labor at zero wage rates.

2. *The wages are determined by equations (1), (2) and (3);*

3. *The government budget constraint holds:* $\tau_t = M_t - M_{t-1}$;

4. *Markets clear:* $\sum_{i=1,2,3} c_t^i = (1-n_t)^\alpha$; $\sum_{i=1,2,3} k_t^i = n_t^\alpha$; $\sum_{i=1,2,3} m_t^i = M_t$

In any equilibrium, the following accounting identity must hold:

$$p_t^k n_t^\alpha + p_t^c (1-n_t)^\alpha = M_t = w_t^1 + w_t^2 + w_t^3,$$

where the left side is nominal GDP and the right side is national income. With Cobb-Douglas preferences $u(c,k) = c^{1-\phi}k^\phi$, household optimization implies:

$$p_t^k k_t^i = \phi \left[w_{t-1}^i + \frac{\tau_t}{3} \right] \text{ and } p_t^c c_t^i = (1-\phi) \left[w_{t-1}^i + \frac{\tau_t}{3} \right]$$

Since old households spend their entire nominal wealth M_t on goods and services:

$$p_t^k n_t^\alpha = \phi M_t \text{ and } p_t^c (1-n_t)^\alpha = (1-\phi) M_t$$

which implies:

$$\frac{p_t^k}{p_t^c} = \frac{\phi}{1-\phi} \frac{F(1-n_t)}{n_t^\alpha}$$

From (3):

$$\frac{p_t^k}{p_t^c} = \frac{n_t}{1-n_t} \frac{F(1-n_t)}{n_t^\alpha}$$

Combining the previous two equations yields:

$$n_t = \phi$$

so that,

$$\frac{p_t^k}{p_t^c} = \frac{\phi}{1-\phi} \frac{F(1-\phi)}{\phi} \equiv \xi(\phi)$$

2.1.1 A labor supply–labor demand representation

The closed-economy equilibrium allocation of the mobile factor n_t admits a simple graphical characterization that will be useful later.

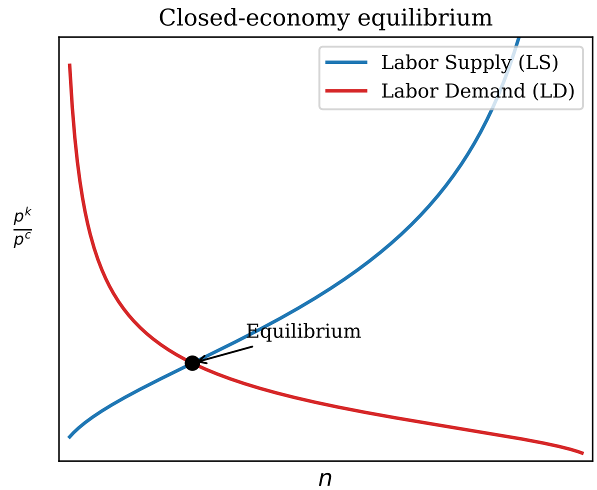


Figure 1: Closed-economy equilibrium as the intersection of a labor supply schedule (4) (blue curve; sectoral choice of the mobile factor) and a labor demand schedule (5) (red curve; expenditure shares). The horizontal axis is the mobile labor share in the goods sector, n, and the vertical axis is the relative price p^k/p^c.

Labor supply (sectoral choice). The mobile factor (type 3) is indifferent across sectors in equilibrium. Using $n^\alpha = n^\alpha$, the no-arbitrage condition $w_t^3 = p_t^k F'(n_t) = p_t^c F'(1-n_t)$ implies

$$\frac{p_t^k}{p_t^c} = \left(\frac{1-n_t}{n_t}\right)^{\alpha-1}. \tag{4}$$

We interpret (4) as a *labor supply schedule* to the goods (tradables) sector: a higher relative price p_t^k/p_t^c raises the relative return to working in the goods sector and increases n_t.

Labor demand (expenditure shares). Using old-age expenditure shares, $p_t^k n_t^\alpha = \phi M_t$ and $p_t^c F(1-n_t) = (1-\phi)M_t$. Combining these conditions yields

$$\frac{p_t^k}{p_t^c}\,\frac{n_t^\alpha}{(1-n_t)^\alpha} = \frac{\phi}{1-\phi}. \tag{5}$$

We interpret (5) as a *labor demand schedule* for the goods sector: for a given n_t, the relative price must adjust so that households devote fraction ϕ of nominal spending to goods.

Equilibrium. The intersection of (4) and (5) delivers $n_t = \phi$.

From nominal GDP we can derive prices:

$$p_t^c = \frac{M_t}{\xi(\phi)\,\phi^\alpha + (1-\phi)^\alpha}$$
$$p_t^k = \frac{\xi(\phi)\,M_t}{\xi(\phi)\,\phi^\alpha + (1-\phi)^\alpha}$$

Sector prices grow at rate μ. The aggregate price level is

$$p_t = \phi p_t^k + (1-\phi)\,p_t^c$$

and the equilibrium inflation rate is μ.

Real allocations in this closed economy are invariant to inflation. There is no savings decision and no portfolio allocation problem; inflation scales all nominal wages and prices uniformly. What inflation does is finance the lump-sum transfers to the old—that is its only role here.

2.2 Open Economy

Now add a second country, identical to the first in every respect except one: it has no financial assets. No money, no private financial markets. In autarky this foreign economy cannot function—there is no medium of exchange and hence no production. Open international trade creates an opportunity. The foreign economy has labor and technology; the home economy has the only transaction asset.

What do we expect when trade opens? Goods are tradable; services are not. Home and foreign tradables are perfect substitutes (we relax this in Section 4.1), so the real exchange rate between them is fixed at par. There is a single currency, so there is no nominal exchange rate to worry about.

Let foreign allocations be denoted with an asterisk $(*)$. Let p_t^c and p_t^{c*} denote the price of services at home and abroad. Goods are tradable with a common price p_t^k.

Definition 2. *An open economy competitive equilibrium is a sequence of consumption allocations and money holdings in domestic and foreign countries* $\{(c_t^i, k_t^i, m_t^i), (c_t^{i*}, k_t^{i*}, m_t^{i*})\}_{t=0, i\in\{1,2,3\}}^\infty$, *the fraction of type 3 households in the goods sector* $\{n_t, n_t^*\}_{t=0}^\infty$, *prices* $\{p_t^c, p_t^{c*}, p_t^k\}_{t=0}^\infty$, *wages* $\{w_t^i, w_t^{i*}\}_{t=0, i\in\{1,2,3\}}^\infty$, *and government policy* $\{M_t, \tau_t\}_{t=0}^\infty$, *such that*

1. *Given prices, wages and government policy, allocation* $\{c_t^i, k_t^i, m_t^i\}_{t=0}^\infty$ *solves the problem of domestic household* $i = 1, 2, 3$: $\max u\left(c_{t+1}^i, k_{t+1}^i\right)$ *subject to:*

$$m_t^i \le w_t^i \text{ and } p_{t+1}^c c_{t+1}^i + p_{t+1}^k k_{t+1}^i \le m_t^i + \frac{\tau_{t+1}}{3}$$

2. *Given prices, wages and government policy, allocation* $\{c_t^{i*}, k_t^{i*}, m_t^{i*}\}_{t=0}^{\infty}$ *solves the problem of foreign household* i *(the difference is that they do not receive the transfer):* $\max u\left(c_{t+1}^{i*}, k_{t+1}^{i*}\right)$*, subject to:*

$$m_t^{i*} \leq w_t^{i*} \text{ and } p_{t+1}^{c*} c_{t+1}^{i*} + p_{t+1}^{k} k_{t+1}^{i*} \leq m_t^{i*}$$

3. *Domestic wages are determined according to equations (1), (2) and (3) (and similarly for foreign wages).*

4. *The government budget constraint holds:* $\tau_t = M_t - M_{t-1}$*;*

5. *Markets clear*

$$\sum_{i=1,2,3} c_t^i = (1-n_t)^\alpha$$
$$\sum_{i=1,2,3} c_t^{i*} = (1-n_t^*)^\alpha$$
$$\sum_{i=1,2,3} k_t^i + \sum_{i=1,2,3} k_t^{i*} = n_t^\alpha + n_t^{*\alpha}$$
$$\sum_{i=1,2,3} m_t^i + \sum_{i=1,2,3} m_t^{i*} = M_t$$

The equilibrium can be reduced to a system of three equations, as the following lemma states.

Lemma 1. *In the open economy, competitive equilibrium is characterized by the following system of equations.*

$$(1-n_t^*)\, n_t^{*\alpha-1} = (1-\phi)\, \frac{p_{t-1}^k}{p_t^k} n_{t-1}^{*\alpha-1} \tag{6}$$
$$(1-n_t)\, n_t^{\alpha-1} = (1-\phi)\, \frac{p_{t-1}^k}{p_t^k} \left[\mu n_{t-1}^{\alpha-1} + (\mu-1)\, n_{t-1}^{*\alpha-1}\right] \tag{7}$$
$$n_t^\alpha + n_t^{*\alpha} = \phi \frac{p_{t-1}^k}{p_t^k} \mu \left[n_{t-1}^{\alpha-1} + n_{t-1}^{*\alpha-1}\right] \tag{8}$$
$$\tag{9}$$

and

$$p_0^c\,(1-n_0)^\alpha = (1-\phi)\, M_0 \tag{10}$$
$$p_0^k\,(n_0^{*\alpha} + n_0^\alpha) = \phi M_0 \tag{11}$$

$$\frac{p_0^k}{p_0^c} = \frac{n_0}{1-n_0}\frac{(1-n_0)^\alpha}{n_0^\alpha} \tag{12}$$

with M_0 given and $n_0^* = 1$

Proof. See Appendix A.1. □

Interpretation of the equilibrium conditions. Equation (6) is the *foreign service-market clearing condition*: the value of services produced abroad equals the fraction $(1-\phi)$ of foreigners' nominal income, with the ratio p_{t-1}^k/p_t^k capturing goods-price inflation. This equation pins down n_t^* given the history (n_{t-1}^*) and the price path.

Equation (7) is the *home service-market clearing condition.* Home residents spend fraction $(1-\phi)$ of their total nominal income on services. The term μ multiplying home wages reflects money injected each period (seigniorage), while $(\mu - 1)$ times foreign wages captures seigniorage collected from foreign money holdings. This extra income is the quantitative heart of the exorbitant privilege.

Equation (8) is *global goods-market clearing*: world tradables production equals world demand, where demand is fraction ϕ of world nominal income (scaled by μ to account for new money).

Together, the three equations determine the sequence $(n_t, n_t^*, p_{t-1}^k/p_t^k)$, tracing out how the global labor allocation and price path co-evolve.

The opening period is especially informative. Home's initial old hold all the money; they can buy goods and services, and production proceeds normally in both sectors. The foreign old have nothing—no money, no claims—and can buy nothing. Since foreign services cannot be traded and no one in the foreign economy can pay for them, no services are produced there in period 0. Every mobile worker in the foreign economy goes into goods production; type 2 workers—specific to services—are idle. The entire foreign goods output is shipped to the home economy in exchange for money. This is the transaction that sets everything in motion, and it is why the initial condition is $n_0^* = 1$. Combining the initial conditions gives

$$\frac{n_0}{1-n_0}\frac{1+n_0^\alpha}{n_0^\alpha} = \frac{\phi}{1-\phi}$$

Importantly, n_0 does not depend on μ. And $n_0 < \phi$: the immediate impact of opening trade is a contraction of home goods production, as the flood of cheap foreign goods lowers the relative return to producing tradables at home.

Given the initial labor allocation, the initial prices are:

$$p_0^c = \frac{(1-\phi)\, M_0}{(1-n_0)^\alpha}$$
$$p_0^k = \frac{\phi M_0}{1+n_0^\alpha}$$

Let m_t^d and m_t^{d*} be the money that young households hold at the end of period t, with $m_t^d + m_t^{d*} = M_t = \mu^t M_0$. In the initial period:

$$m_0^d = \frac{\phi M_0 n_0^\alpha}{1+n_0^\alpha} + (1-\phi)\, M_0$$

and

$$m_0^{d*} = \frac{\phi M_0}{1+n_0^\alpha}$$

Let GDP_t and GDP_t^* denote nominal GDP in the home and foreign economies, and EXP_t and EXP_t^* aggregate nominal expenditure (total spending by the old, inclusive of seigniorage transfers):

$$\mathrm{GDP}_t = p_t^k n_t^\alpha + p_t^c\,(1-n_t)^\alpha$$
$$\mathrm{GDP}_t^* = p_t^k n_t^{*\alpha} + p_t^{c*}\,(1-n_t^*)^\alpha$$

$$\mathrm{EXP}_t = m_{t-1}^d + (\mu-1)\left(m_{t-1}^d + m_{t-1}^{d*}\right)$$
$$\mathrm{EXP}_t^* = m_{t-1}^{d*}$$

2.2.1 Steady State

Define a steady state as a situation where $n_t = n$, $n_t^* = n^*$ and $p_t^c/p_{t-1}^c = p_t^{c*}/p_{t-1}^{c*} = p_t^k/p_{t-1}^k = \mu$.

Three properties of the steady state are worth stating before we prove them. First, the reserve-currency asymmetry reallocates mobile labor in opposite directions in the two countries: the foreign economy tilts toward tradables and the home economy toward nontradables. Second, this reallocation changes the relative wage of goods and service producers in opposite directions across countries. Third, the home economy runs a persistent trade deficit. These are not separate phenomena; they are all consequences of the same underlying force: foreign demand for the home transaction asset.

Let us define the wage ratio as the wage of a goods producer relative to that of a service producer.

Proposition 1. *Suppose $\mu \geq 1$. In steady state, the following statements are true:*

1- $n^* \geq \phi \geq n$, with equality if and only if $\mu = 1$;

2- $\frac{w_t^1}{w_t^2} \leq \frac{\phi}{1-\phi} \leq \frac{w_t^{1*}}{w_t^{2*}}$, with equality if and only if $\mu = 1$;

3- $\dfrac{\mathrm{EXP}_t}{\mathrm{GDP}_t} \geq 1 \geq \dfrac{\mathrm{EXP}_t^*}{\mathrm{GDP}_t^*}$, with equality if and only if $\mu = 1$.

Proof. See Appendix A.2. □

As long as money grows, the foreign economy produces more tradables than in autarky and ships the excess home in exchange for money. The home economy absorbs those shipments and tilts its mobile labor toward nontradables. The goods-to-services wage ratio falls at home and rises abroad. Trade never balances as long as money is growing: the home economy always consumes more than it produces, paying with claims on future seigniorage. Even if money growth stops, the accumulated foreign money balances remain outstanding—the debt is never paid, it is simply held.

The foreign economy tilts toward tradables because tradable goods are the only object it can deliver in exchange for money. The home economy tilts toward nontradables because access to cheap imported goods lowers the relative profitability—and the relative wage—of producing tradables at home, drawing mobile labor into the nontradables sector. The two effects are the same phenomenon seen from opposite sides.

2.2.2 Steady-state labor supply and labor demand

The steady-state reallocation $n < \phi < n^*$ has a transparent interpretation in terms of a labor-supply/labor-demand diagram, analogous to Figure 1.

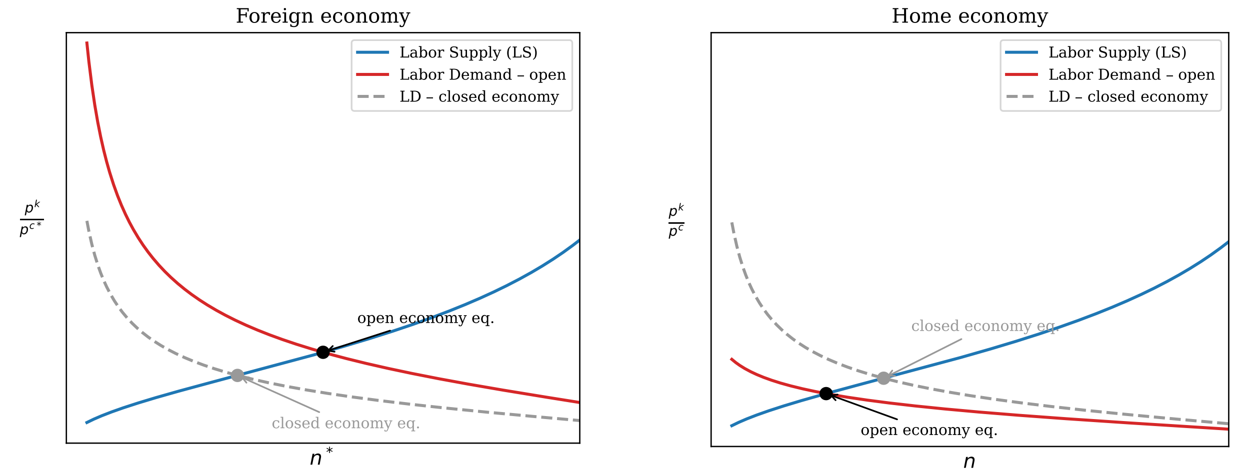
Labor supply (sectoral choice). In both countries, the mobile factor must be indifferent across sectors. With $n^\alpha = n^\alpha$, this delivers the same upward-sloping "labor supply" schedule as in the closed economy:

$$\frac{p^k}{p^c} = \left(\frac{1-n}{n}\right)^{\alpha-1}, \qquad \frac{p^k}{p^{c*}} = \left(\frac{1-n^*}{n^*}\right)^{\alpha-1}. \tag{13}$$

Labor demand (asset-demand wedge). What differs across countries is the schedule linking relative prices to expenditure shares once the reserve-currency asymmetry is taken into account.

In the foreign economy, steady-state goods-market clearing and the fact that prices grow at rate μ imply the "labor demand" condition

$$\frac{(1-n^*)^\alpha}{n^{*\,\alpha-1}} = \frac{1-\phi}{\mu}\,\frac{p^k}{p^{c*}}. \tag{14}$$



(a) Foreign economy.

(b) Home economy.

Figure 2: Open-economy steady state as an intersection of labor supply (blue curve; (13)) and labor demand (red curve; (14)–(15)). The dashed gray curve is the closed-economy labor demand (5), and the gray dot is the closed-economy equilibrium. The reserve-currency asymmetry shifts the foreign labor-demand schedule up (raising n^* above the closed-economy benchmark) and shifts the home labor-demand schedule down (lowering n below the closed-economy benchmark).

Because the foreign economy must acquire the transaction asset by exporting tradables, the shadow value of producing tradables is high, shifting the foreign labor-demand curve up relative to autarky.

In the home economy, the analogous steady-state condition becomes

$$\frac{p^k}{p^c} = \frac{\phi}{1-\phi} \frac{(1-n)^\alpha}{\left(1-\frac{1}{\mu}\right) n^{*\,\alpha-1} + n^\alpha}. \tag{15}$$

The term $\left(1-\frac{1}{\mu}\right) n^{*\,\alpha-1}$ captures the additional nominal spending power accruing to home residents from money growth on a balance sheet partly held abroad—seigniorage from foreign asset demand. This shifts the home labor-demand curve down, reducing the relative price of tradables and drawing mobile labor into nontradables.

2.2.3 Transition

The steady-state results describe long-run allocations. The transition dynamics, which we cannot characterize analytically, are illustrated in Figure 3 for the parameter values $\phi = 0.2$, $\alpha = 0.5$, $\mu = 1.05$.

The opening period is the most extreme part of the transition. The foreign economy, eager to

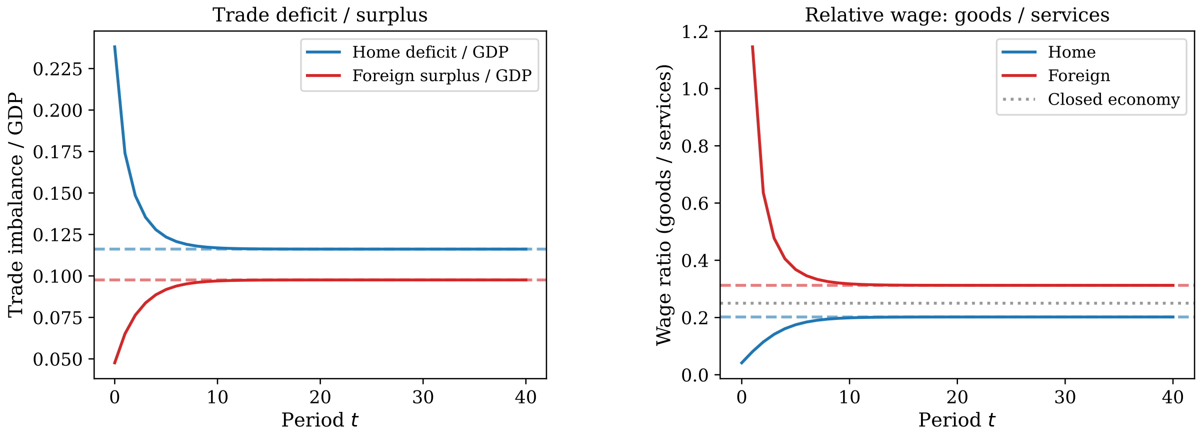


Figure 3: Transition dynamics following the opening of trade ($\phi = 0.2$, $\alpha = 0.5$, $\mu = 1.05$). Left panel: home trade deficit as a share of home goods GDP (blue, declining from an initial peak toward steady state) and foreign trade surplus as a share of foreign goods GDP (red, rising toward steady state). Right panel: wage ratio of goods producers to service producers for the home economy (blue) and foreign economy (red); the dotted gray line marks the common closed-economy benchmark. In the open-economy steady state the home ratio lies below the autarky level (deindustrialization) while the foreign ratio lies above it.

acquire money the moment trade opens, exports its entire goods output to the home economy. The trade deficit jumps immediately to its maximum. If there is no money growth, the foreign economy has all the money it needs after one period and stops exporting; the deficit falls to zero and the economies thereafter operate as two closed economies, the foreign country holding its money balances forever.

With money growth, the foreign economy must continuously export to replenish the real value of money holdings eroded by inflation. The economies transition gradually to the steady state described above. The trade deficit declines from its initial peak but remains positive throughout. For the transition figure, we report the trade imbalance relative to goods GDP. The goods-to-services wage ratio in the home economy follows the same pattern in reverse: it is lowest on impact and rises toward its steady-state value.

3 Tariff Policy

To connect the model to the policy discussion, interpret the home economy as the United States and the foreign economy as the rest of the world. The open-economy equilibrium just described features a structural trade deficit: the dollar is "overvalued" in the sense that it commands more real goods than it would if the U.S. were not the reserve-currency issuer,[3] and the home tradables sector has contracted relative to the closed-economy benchmark. Both phenomena follow from the same source: foreign demand for U.S.-issued transaction assets. This is what Miran Miran (2024) identifies as the mechanism behind the persistent U.S. trade deficit.

The policy question is whether a unilateral import tariff can undo this. We model the tariff as a proportional tax θ on goods produced abroad and consumed at home. Given perfect substitutability, all trade flows from the foreign economy to the home economy; without loss of generality, foreign consumers consume only foreign-produced goods. Tariff revenue is rebated lump-sum to home old-age consumers.

Let p_t^k and p_t^{k*} be the prices of home- and foreign-produced goods. Let k_{Ht} and k_{Ft} denote home consumption of domestic and foreign goods, and k_{Ft}^* consumption of foreign goods in the foreign economy. In equilibrium $p_t^k = (1+\theta)\, p_t^{k*}$.

Definition 3. *A tariff distorted competitive equilibrium is a sequence of domestic and foreign allocations and money holdings* $\{(c_t^i, k_{Ht}^i, k_{Ft}^i, m_t^i), (c_t^{i*}, k_{Ft}^{i*}, m_t^{i*})\}_{t=0,\ i\in\{1,2,3\}}^{\infty}$*, mobile-labor allocations* $\{n_t, n_t^*\}_{t=0}^{\infty}$*, prices* $\{p_t^c, p_t^{c*}, p_t^k, p_t^{k*}\}_{t=0}^{\infty}$*, wages* $\{w_t^i, w_t^{i*}\}_{t=0,\ i\in\{1,2,3\}}^{\infty}$*, and government policy* $\{\theta, M_t, \tau_t\}_{t=0}^{\infty}$*, such that*

1. *Given prices, wages and government policy, allocation* $\{c_t^i, k_t^i, m_t^i\}_{t=0}^{\infty}$ *solves the problem of domestic household* i*':* $\max u\left(c_{t+1}^i, k_{Ht+1}^i + k_{Ft+1}^i\right)$*, subject to,*

$$m_t^i \le w_t^i \ \textit{and}\ p_{t+1}^c c_{t+1}^i + p_{t+1}^k k_{Ht+1}^i + (1+\theta)\, p_{t+1}^{k*} k_{Ft+1}^i \le m_t^i + \frac{\tau_{t+1}}{3};$$

2. *Given prices, wages, and government policy, allocation* $\{c_t^{i*}, k_t^{i*}, m_t^{i*}\}_{t=0}^{\infty}$ *solves the problem of the foreign individual worker* i*' (note: the difference is that they do not receive the transfer):* $\max u\left(c_{t+1}^{i*}, k_{Ft+1}^{i*}\right)$ *subject to,*

$$m_t^{i*} \le w_t^{i*} \ \textit{and}\ p_{t+1}^{c*} c_{t+1}^{i*} + p_{t+1}^{k*} k_{Ft+1}^{i*} \le m_t^{i*}$$

3. *Domestic wages are determined according to equations (1), (2) and (3) (and similarly for foreign wages).*

4. *Government budget constraint holds:*

$$\tau_t = M_t - M_{t-1} + \theta p_t^{k*} \left(\sum_{i=1,2,3} k_{Ft}^i\right)$$

[3] Dollar "overvaluation" here takes the form of a lower price level since there is no nominal exchange rate.

5. *Markets clear:*

$$\sum_{i=1,2,3} c_t^i = (1-n_t)^\alpha$$
$$\sum_{i=1,2,3} c_t^{i*} = (1-n_t^*)^\alpha$$
$$\sum_{i=1,2,3} k_{Ht}^i = n_t^\alpha$$
$$\sum_{i=1,2,3} k_{Ft}^i + \sum_{i=1,2,3} k_{Ft}^{i*} = n_t^{*\alpha}$$
$$\sum_{i=1,2,3} m_t^i + \sum_{i=1,2,3} m_t^{i*} = M_t$$

The equilibrium conditions under the tariff are summarized in the following lemma.

Lemma 2. *Equilibrium allocations and prices in the open economy with a tariff are characterized by the following system of equations for* $t > 0$*:*

$$(1-n_t^*)\, n_t^{*\alpha-1} = (1-\phi)\, \frac{p_{t-1}^{k*}}{p_t^{k*}} n_{t-1}^{*\alpha-1} \tag{16}$$

$$\begin{aligned}(1-n_t)\, n_t^{\alpha-1} = (1-\phi) \Bigg[\mu \frac{p_{t-1}^{k}}{p_t^{k}} n_{t-1}^{\alpha-1} + (\mu-1)\, \frac{p_{t-1}^{k*}}{p_t^{k}} n_{t-1}^{*\alpha-1} \\ + \theta \frac{p_t^{k*}}{p_t^{k}} \left(n_t^{*\alpha} - \phi \frac{p_{t-1}^{k*}}{p_t^{k*}} n_{t-1}^{*\alpha-1} \right) \Bigg]\end{aligned} \tag{17}$$

$$\begin{aligned}n_t^\alpha + n_t^{*\alpha} = \phi \Bigg[\mu \frac{p_{t-1}^{k}}{p_t^{k}} n_{t-1}^{\alpha-1} + (\mu-1)\, \frac{p_{t-1}^{k*}}{p_t^{k}} n_{t-1}^{*\alpha-1} \\ + \theta \frac{p_t^{k*}}{p_t^{k}} \left(n_t^{*\alpha} - \phi \frac{p_{t-1}^{k*}}{p_t^{k*}} n_{t-1}^{*\alpha-1} \right) \Bigg] + \phi \frac{p_{t-1}^{k*}}{p_t^{k*}} n_{t-1}^{*\alpha-1}\end{aligned} \tag{18}$$

and

$$p_t^k = (1+\theta) p_t^{k*}$$

Proof. See Appendix A.3. □

The key observation is equation (16): it is identical to the no-tariff foreign condition (6). The tariff does not appear in the foreign equilibrium condition at all. This is not a coincidence. The foreign economy's decisions—how much to produce, how much to export, and how much money to hold—are governed entirely by its demand for the home transaction asset. None of these margins depend on what the home country charges at the border. As a result, θ drops out of the foreign steady state entirely.

In steady state, imposing the steady-state condition in equation (16) gives

$$n^* = \frac{\mu - 1}{\mu} + \frac{\phi}{\mu}.$$

This is independent of the tariff rate. Imposing a tariff on imports does not affect the foreign steady-state allocation.

Turning to the home economy: imposing the steady-state condition and substituting $p_t^k = (1+\theta)p_t^{k*}$ in equation (17) and simplifying (substituting for n^* and rearranging) yields

$$\phi n^{\alpha-1} - n^{\alpha} = (1-\phi)\left(\frac{\mu - 1}{\mu}\right) n^{*\alpha-1}$$

This too is independent of θ. The home labor allocation is also invariant to the tariff. Long-run sectoral employment and output are unchanged.

Let $K_F \equiv \sum_{i=1,2,3} k_{Ft}^i$ denote aggregate home imports of foreign-produced goods.

Proposition 2 (Long-Run Tariff Neutrality)**.** *Suppose $\mu > 1$. For any tariff rate $\theta \geq 0$, the steady-state allocation (n, n^*, K_F) is invariant to θ. In particular:*

(i) $n^* = 1 - (1-\phi)/\mu$*, independently of* θ*;*

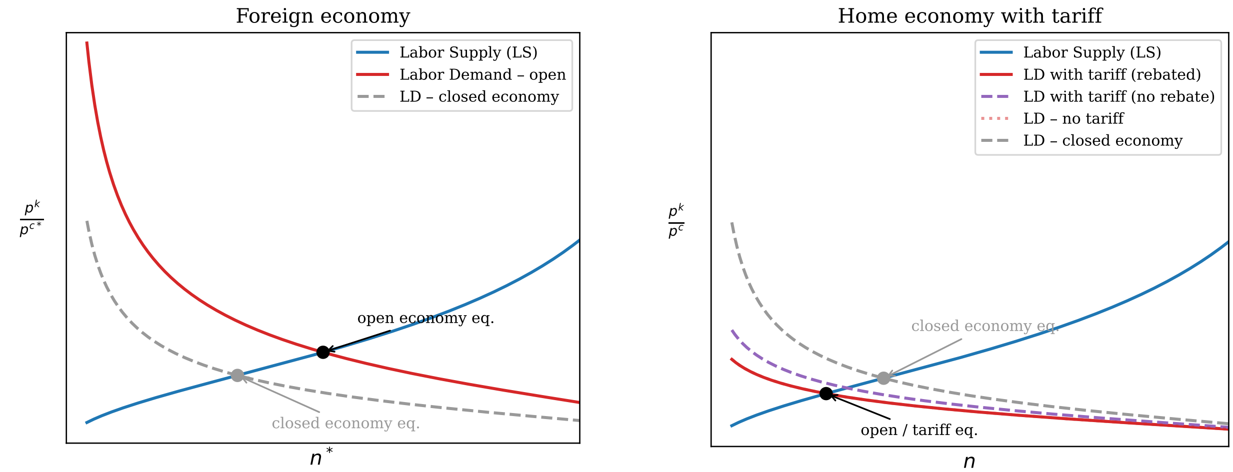
(ii) n *satisfies* $\phi n^{\alpha-1} - n^{\alpha} = (1-\phi)\big[(\mu-1)/\mu\big]n^{*\,\alpha-1}$*, independently of* θ*;*

(iii) $K_F = \big[(\mu-1)/\mu\big]n^{*\,\alpha-1}$*, independently of* θ*.*

Proof. See Appendix A.4. □

The neutrality result follows directly from the two sides of the model. The foreign economy must export tradables to acquire home money; the quantity it exports is determined by its need for the transaction asset, not by the tariff wedge. Formally, foreign steady-state behavior is governed solely by the service-market clearing condition (16), which does not contain θ. This pins down n^* (and hence K_F) independently of the tariff. At home, the tariff raises the consumer price of imported goods relative to the producer price, but—because tariff revenue is rebated lump-sum to home old-age consumers—the extra spending power exactly offsets the price increase. The home labor market therefore sees no change in the real return to tradables production, leaving n unchanged.

The tariff cannot eliminate the trade deficit because the deficit is the equilibrium counterpart of foreign demand for the home transaction asset, not an outcome of distorted prices.



(a) Foreign economy.

(b) Home economy.

Figure 4: Steady-state effects of an import tariff. Colors: the blue curve is labor supply (sectoral choice), the red curve is labor demand, and the dashed gray curve is the closed-economy labor demand with the gray dot denoting the closed-economy equilibrium. In the home panel, the purple dashed curve is labor demand in the tariff economy without the lump-sum tariff rebate. Despite shifts in home labor demand, the equilibrium mobile labor allocation n is unchanged relative to the no-tariff open-economy benchmark.

3.1 A labor supply–labor demand view of tariffs

In steady state, the foreign economy is unaffected by the home import tariff: the foreign labor supply schedule is unchanged and the foreign labor demand condition continues to pin down n^* as in Proposition 1. In the home economy, the tariff modifies the relationship between relative prices and expenditure shares (because tariff revenue is rebated lump-sum to home old-age consumers). Combining steady-state service demand with $p^k = (1+\theta)p^{k*}$ yields the home "labor demand" schedule under a tariff:

$$\frac{p^k}{p^c} = \frac{\phi}{1-\phi}\,\frac{(1-n)^\alpha}{\left(1-\frac{1}{\mu}\right)\frac{1}{1+\theta}\,n^{*\,\alpha-1} + \frac{\theta}{1+\theta}\left(n^{*\,\alpha} - \frac{\phi}{\mu}n^{*\,\alpha-1}\right) + n^\alpha} \qquad \text{(steady state).}$$

Figure 4 illustrates the key point: although the tariff shifts the home labor-demand schedule, the intersection with the unchanged labor-supply schedule occurs at the same n. The tariff therefore does not "reindustrialize" the home economy in the long run.

3.2 Valuation Effects and Trade-Deficit Measurement

What the tariff does change is relative prices. Home-produced goods are always more expensive than foreign-produced goods by the factor $(1+\theta)$. A higher tariff therefore compresses the foreign producer price relative to the home consumer price. Since import quantities are unchanged, this price wedge is the only channel through which the measured trade deficit can move.

Denote foreign demand for foreign-produced goods by K^*_{Ft}. Then

$$K^*_{Ft} = \phi \frac{p^{k*}_{t-1}}{p^{k*}_t} n^{*\alpha-1}_{t-1}$$

and the amount of goods imported by the home economy is

$$K_{Ft} = n^{*\alpha}_t - K^*_{Ft} = n^{*\alpha}_t - \phi \frac{p^{k*}_{t-1}}{p^{k*}_t} n^{*\alpha-1}_{t-1}$$

In the benchmark model, home exports are zero, so the trade deficit equals import spending valued at world prices. Therefore the trade deficit as a share of GDP can be written as

$$\frac{\text{EXP}_t}{\text{GDP}_t} - 1 = \frac{p^{k*}_t K_{Ft}}{p^k_t n^\alpha_t + p^c_t \left(1 - n_t\right)^\alpha}.$$

The trade deficit as a share of GDP is

$$\frac{\text{EXP}_t}{\text{GDP}_t} - 1 = \frac{p^{k*}_t}{p^k_t} \frac{n^{*\alpha}_t - \phi \frac{p^{k*}_{t-1}}{p^{k*}_t} n^{*\alpha-1}_{t-1}}{\frac{n^\alpha_t}{n_t}}$$

In steady state (after substituting for n^*):

$$\frac{\text{EXP}_t}{\text{GDP}_t} - 1 = \frac{1}{1+\theta} \left(\frac{\mu - 1}{\mu}\right) n^{*\alpha-1} \frac{n}{n^\alpha}.$$

The measured trade deficit falls as the tariff rises. But the decline is entirely mechanical. The import quantity K_F is pinned down by foreign production and demand—by μ, ϕ, and α—and is invariant to θ. What changes is the price at which imports enter the trade-balance calculation: a higher tariff depresses $p^{k*}/p^k = 1/(1+\theta)$, and since imports are valued at world prices while home GDP is valued at domestic prices, the ratio falls. This is a valuation effect, not a quantity effect. The home economy is not importing less; it is simply valuing its imports more cheaply relative to its own output.

3.3 Burden Sharing and Incidence

The long-run picture is this: quantities are unchanged, but prices are not. The tariff drives a wedge between what home consumers pay and what foreign producers receive. Both sides of this wedge are borne by someone.

Foreign exporters receive lower net-of-tariff prices—a terms-of-trade deterioration. Home consumers pay more for imported tradables. The home government collects the difference. In this sense the tariff looks more like an international income transfer than a border protection device: it shifts resources from foreign producers and home consumers to the home fiscal authority, without altering the real quantities that cross the border.

To see why incidence is shared, note that in equilibrium $p^k = (1+\theta)p^{k*}$. A higher tariff either raises the home consumer price p^k, compresses the foreign producer price p^{k*}, or some combination of both. In our model, both adjust, so both home consumers and foreign producers bear part of the wedge. This shared-incidence pattern aligns with the empirical findings in Amiti et al. (2019) and Fajgelbaum et al. (2020), and with the broader message on valuation channels in Caliendo et al. (2025) and Itskhoki and Mukhin (2025).

4 Extensions

The benchmark results rest on several simplifications worth relaxing. First, we assumed home and foreign tradables are perfect substitutes, so demand elasticity played no role. Second, we compared tariffs to the counterfactual of no policy; the natural alternative is a direct production subsidy to the tradables sector. Third, we assumed tariff revenue is rebated lump-sum; alternative fiscal arrangements may modify the neutrality logic. Fourth, the benchmark features a single transaction asset with no competing store of value. We take up each in turn.

4.1 Imperfect substitution between home and foreign tradables

Consider the following preferences in the home economy:

$$\nu\left(k_{Ht}, k_{Ft}, c_t\right) = \left[(1-\eta)^{\frac{1}{\rho}}\, k_{Ht}^{\frac{\rho-1}{\rho}} + \eta^{\frac{1}{\rho}} k_{Ft}^{\frac{\rho-1}{\rho}}\right]^{\frac{\phi\rho}{\rho-1}} c_t^{1-\phi}, \quad 0 < \rho < \infty.$$

where k_{Ht} and k_{Ft} are demands for domestic and foreign-produced tradables in the domestic economy and ρ is the substitution elasticity. The limit $\rho \to \infty$ corresponds to our baseline (perfect substitutes).

For the foreign economy, assume

$$\nu\left(k^*_{Ft}, c^*_t\right) = k^{*\phi}_{Ft} c^{*1-\phi}_t.$$

As before, foreign consumers consume only foreign-produced tradables.

Home CES demand system: full derivation

Fix a date $t+1$ and suppress time subscripts where convenient. Home old-age tradables consumption is a composite of home and foreign varieties,

$$K \equiv \left[(1-\eta)^{\frac{1}{\rho}} K_H^{\frac{\rho-1}{\rho}} + \eta^{\frac{1}{\rho}} K_F^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}},$$

and period utility over the goods composite and services is

$$U = K^{\phi} c^{1-\phi}.$$

Given composite expenditure on tradables, $E_K \equiv p^k K_H + (1+\theta) p^{k*} K_F$, cost minimization implies the standard CES price index

$$P^k \equiv \left[(1-\eta)\left(p^k\right)^{1-\rho} + \eta\left((1+\theta)p^{k*}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}.$$

so that $E_K = P^k K$. The associated Hicksian (and, with homothetic preferences, Marshallian) demands are

$$K_H = (1-\eta)\left(\frac{p^k}{P^k}\right)^{-\rho} K,$$
$$K_F = \eta\left(\frac{(1+\theta)p^{k*}}{P^k}\right)^{-\rho} K.$$

Equivalently, the expenditure shares are

$$\frac{p^k K_H}{E_K} = \frac{(1-\eta)\left(p^k\right)^{1-\rho}}{(1-\eta)\left(p^k\right)^{1-\rho} + \eta\left((1+\theta)p^{k*}\right)^{1-\rho}},$$
$$\frac{(1+\theta)p^{k*} K_F}{E_K} = \frac{\eta\left((1+\theta)p^{k*}\right)^{1-\rho}}{(1-\eta)\left(p^k\right)^{1-\rho} + \eta\left((1+\theta)p^{k*}\right)^{1-\rho}}.$$

Finally, Cobb–Douglas between K and c implies $E_K = \phi E$ and $p^c c = (1-\phi)E$, where E denotes total nominal expenditure by the home old at date $t+1$.

The demand in the foreign economy is

$$
\begin{aligned}
p^{c*}_{t+1} c^*_{t+1} &= (1-\phi)\, p^{k*}_t n^{*\alpha-1}_t \\
p^{k*}_{t+1} K^*_{Ft+1} &= \phi p^{k*}_t n^{*\alpha-1}_t
\end{aligned}
$$

with optimal sector choice

$$
\frac{p^{c*}_t}{p^{k*}_t} = \left(\frac{n^*_t}{1-n^*_t}\right)^{\alpha-1}. \tag{19}
$$

The demand in the home economy is

$$
\begin{aligned}
p^c_{t+1} c_{t+1} &= (1-\phi)\left(\mu p^k_t n^{\alpha-1}_t + (\mu-1)\, p^{k*}_t n^{*\alpha-1}_t + \theta p^{k*}_{t+1} k_{Ft+1}\right) \\
p^k_{t+1} K_{Ht+1} &= \frac{1-\eta}{1-\eta+\eta\left((1+\theta)\frac{p^{k*}_{t+1}}{p^k_{t+1}}\right)^{1-\rho}} \phi \left(\mu p^k_t n^{\alpha-1}_t + (\mu-1)\, p^{k*}_t n^{*\alpha-1}_t \right. \\
&\qquad \left. + \theta p^{k*}_{t+1} K_{Ft+1}\right) \\
(1+\theta)\, p^{k*}_{t+1} K_{Ft+1} &= \frac{\eta\left((1+\theta)\frac{p^{k*}_{t+1}}{p^k_{t+1}}\right)^{1-\rho}}{1-\eta+\eta\left((1+\theta)\frac{p^{k*}_{t+1}}{p^k_{t+1}}\right)^{1-\rho}} \phi \left(\mu p^k_t n^{\alpha-1}_t + (\mu-1)\, p^{k*}_t n^{*\alpha-1}_t \right. \\
&\qquad \left. + \theta p^{k*}_{t+1} K_{Ft+1}\right)
\end{aligned}
$$

with optimal sector choice

$$
\frac{p^c_t}{p^k_t} = \left(\frac{n_t}{1-n_t}\right)^{\alpha-1}. \tag{20}
$$

Imposing steady state in the foreign economy yields

$$
\begin{aligned}
n^* &= \frac{\mu-1}{\mu} + \frac{\phi}{\mu} \\
k^*_F &= \frac{\phi}{\mu} n^{*\alpha-1}.
\end{aligned}
$$

Using market clearing for foreign-produced tradables, steady-state home imports are

$$
K_F = n^{*\alpha} - \frac{\phi}{\mu} n^{*\alpha-1} = \left(\frac{\mu-1}{\mu}\right) n^{*\alpha-1}.
$$

As in the benchmark, the long-run import quantity is pinned down by foreign production and foreign demand, invariant to the tariff.

In steady state, the home allocation and the relative price p^k/p^{k*} solve a system of two equations derived from goods-market clearing and the CES demand system:

$$\phi n^{\alpha-1} - n^{\alpha} = (1-\phi)(1+\theta)\frac{p^{k*}}{p^{k}} n^{*\alpha-1}\left(\frac{\mu-1}{\mu}\right) \tag{21}$$

$$n = \frac{(1-\eta)\phi}{(1-\phi)\eta\left((1+\theta)\frac{p^{k*}}{p^{k}}\right)^{1-\rho} + (1-\eta)}. \tag{22}$$

Eliminating $(1+\theta)\frac{p^{k*}}{p^{k}}$ between (21) and (22) shows that the steady-state labor allocation n is independent of θ. Long-run sectoral allocation and outputs are invariant to the tariff even when tradables are imperfect substitutes.

Proposition 3 (CES Tariff Neutrality)**.** *Under one-way CES demand (home consumers substitute between home and foreign tradables with elasticity $\rho > 0$; foreign consumers are specialized in the foreign variety), the steady-state labor allocations (n, n^*) and the import quantity K_F are invariant to the tariff rate θ. The measured trade-deficit-to-GDP ratio falls with θ through the relative price p^{k*}/p^{k}, not through any change in quantities.*

Proof. See Appendix A.5. □

The trade deficit as a share of home GDP in steady state is

$$\frac{p^{k*}K_F}{p^{k}n^{\alpha} + p^{c}(1-n)^{\alpha}} = \frac{p^{k*}}{p^{k}}\frac{n^{*\alpha-1}}{n^{\alpha-1}}\left(\frac{\mu-1}{\mu}\right),$$

which is decreasing in θ through the price ratio p^{k*}/p^{k}. As in the benchmark, the measured deficit falls through valuation rather than through quantity adjustment.

The one-way CES extension above treats the foreign economy as specialized in its own variety. A more symmetric environment allows both countries to substitute between home and foreign tradables (an Armington framework). In that setting the reserve-currency mechanism still pins down the *net* real resource transfer, but tariffs can compress *gross* trade flows and may further shift home production toward nontradables. The formal analysis, including the derivation of equilibrium gross trade flows, the terms-of-trade response to tariffs, and the possibility that home tradables employment falls further with the tariff, is developed in Appendix C. This qualification does not overturn the benchmark result: the tariff still does not undo the reserve-demand mechanism that governs the net transfer.

4.2 Domestic Industrial Policy

Suppose that, instead of imposing a tariff, the home country subsidizes production in the goods-producing (tradables) sector, financed by a lump-sum tax. The decision problem in the foreign country is unchanged.

Home producers in the goods-producing sector receive a subsidy κ per unit of output sold. Since the subsidy is paid to producers rather than levied at the border, the law of one price continues to hold for consumer prices: $p_t^k = p_t^{k*}$. Therefore,

$$\begin{aligned} w_t^1 &= (1+\kappa)\, p_t^k \,(1-\alpha)\, n_t^\alpha \\ w_t^2 &= p_t^c \,(1-\alpha)\,(1-n_t)^\alpha \\ w_t^3 &= (1+\kappa)\, p_t^k \alpha n_t^{\alpha-1} = p_t^c \alpha \,(1-n_t)^{\alpha-1} \end{aligned}$$

Therefore,

$$\frac{p_t^c}{p_t^k} = (1+\kappa)\, n_t^{\alpha-1} \frac{1-n_t}{(1-n_t)^\alpha} = (1+\kappa)\,\frac{n_t^{\alpha-1}}{(1-n_t)^{\alpha-1}} \tag{23}$$

The government budget constraint in the home economy is

$$M_t - M_{t-1} - \kappa p_t^k n_t^\alpha = \tau_t.$$

The demand in the home economy is

$$\begin{aligned} p_{t+1}^c c_{t+1} &= (1-\phi)\left(\mu\,(1+\kappa)\, p_t^k n_t^{\alpha-1} + (\mu-1)\, p_t^{k*} n_t^{*\alpha-1} - \kappa p_{t+1}^k n_{t+1}^\alpha\right) \\ p_{t+1}^k K_{Ht+1} + p_{t+1}^{k*} K_{Ft+1} &= \phi\left(\mu\,(1+\kappa)\, p_t^k n_t^{\alpha-1} + (\mu-1)\, p_t^{k*} n_t^{*\alpha-1} - \kappa p_{t+1}^k n_{t+1}^\alpha\right) \end{aligned} \tag{24}$$

and in the foreign economy

$$\begin{aligned} p_{t+1}^{c*} c_{t+1}^* &= (1-\phi)\, p_t^{k*} n_t^{*\alpha-1} \\ p_{t+1}^{k*} K_{Ft+1}^* &= \phi p_t^{k*} n_t^{*\alpha-1} \end{aligned} \tag{25}$$

along with optimal sector choice condition (19).

Imposing steady state in equation (25) and using optimal sectoral condition (19):

$$\begin{aligned} n^* &= \frac{\mu-1}{\mu} + \frac{\phi}{\mu} \\ k_F^* &= \frac{\phi}{\mu} n^{*\alpha-1} \end{aligned}$$

and from market clearing, imports in the home country are

$$K_F = n^{*\alpha} - \frac{\phi}{\mu} n^{*\alpha-1} = \left(\frac{\mu-1}{\mu}\right) n^{*\alpha-1}$$

Rewriting equation (24) in steady state and eliminating relative prices using equation (23):

$$(1+\kappa)\left(n^{\alpha-1}-n^{\alpha}\right)=(1-\phi)\left((1+\kappa)\,n^{\alpha-1}+\left(\frac{\mu-1}{\mu}\right)n^{*\alpha-1}-\kappa n^{\alpha}\right)$$

which yields

$$(1+\kappa)\,\phi n^{\alpha-1}-(1+\phi\kappa)\,n^{\alpha}=(1-\phi)\left(\frac{\mu-1}{\mu}\right)n^{*\alpha-1}.$$

Unlike in the benchmark and the tariff case, the home labor allocation depends on κ: as subsidies increase, employment and output in the goods-producing sector increase.

Proposition 4 (Industrial Policy)**.** *Suppose $\kappa \geq 0$ is a production subsidy to home tradables. In steady state:*

(i) The foreign labor allocation n^* and the import quantity K_F are invariant to κ.

(ii) The home tradables labor share n is strictly increasing in κ.

Proof. See Appendix A.6. □

The production subsidy succeeds because it operates on a different margin from the tariff. A tariff acts on the *border price of the imported good* and, because the revenue is rebated, has no net effect on the relative return to home tradables production. A production subsidy, by contrast, directly raises the *marginal revenue product of labor in the home goods sector*, shifting the labor supply curve for mobile workers in favor of tradables. Foreign workers face exactly the same conditions as before, so n^* and K_F are unaffected. The takeaway is that the channel through which policy raises n matters: demand-side instruments (tariffs) that merely redistribute income between sectors cannot achieve what supply-side instruments (subsidies) that alter the private return to production can.

Graphical interpretation. The subsidy directly raises the marginal revenue product of labor in the home goods sector. In the labor-supply/labor-demand diagram, this shows up as a shift in the home labor supply schedule (sectoral choice) for the mobile factor, while the demand schedule is unchanged. Figure 5 illustrates how a production subsidy raises the steady-state mobile labor allocation n in the home tradables sector.

The subsidy does not affect the quantity of goods imported. But because the relative wage in the goods sector rises, home goods production increases and home services production falls. The

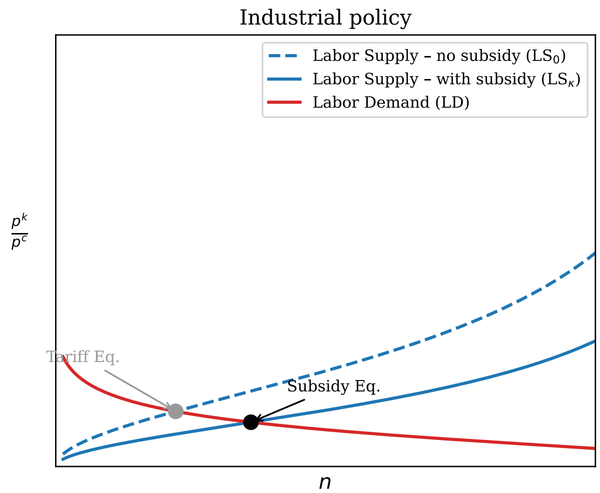


Figure 5: Home-economy steady state with a production subsidy to the goods sector. Colors: the red curve is labor demand, the solid blue curve is labor supply with the subsidy, and the dashed blue curve is labor supply without the subsidy; the gray dot denotes the equilibrium without the subsidy. The subsidy shifts the home labor-supply schedule up, raising the equilibrium mobile labor share in the goods sector n relative to the no-subsidy open-economy benchmark.

trade deficit as a share of GDP is

$$\frac{p_t^{k*} K_{Ft}}{p_t^k n_t^\alpha + p_t^c \left(1 - n_t\right)^\alpha} = \frac{p_t^{k*}}{p_t^k} \frac{\left(\frac{\mu-1}{\mu}\right) n^{*\alpha-1}}{n^\alpha + \frac{p_t^c}{p_t^k} \left(1 - n_t\right)^\alpha}$$

$$= \frac{\left(\frac{\mu-1}{\mu}\right) n^{*\alpha-1}}{\left(1+\kappa\right) n^{\alpha-1} - \kappa n^\alpha}$$

The denominator can be rewritten as

$$\left(1+\kappa\right) n^{\alpha-1} - \kappa n^\alpha = \left(\frac{1-\phi}{\phi}\right)\left(\frac{\mu-1}{\mu}\right) n^{*\alpha-1} + \frac{n^\alpha}{\phi}$$

which is increasing in κ. So, as with the tariff, the subsidy reduces the trade-deficit-to-GDP ratio, despite having no effect on the import quantity. The difference is that under the subsidy, more goods and fewer services are produced at home, and households consume relatively more goods than in the benchmark.

Tariffs and production subsidies are both government interventions in the tradables sector, but they operate on different margins. If the goal is reindustrialization, the instrument has to reach the right margin: the relative return to producing tradables at home.

4.3 Alternative Fiscal Policy

The benchmark analysis assumes tariff revenue is rebated lump-sum to home old-age consumers, leaving μ exogenous. Here we consider an alternative fiscal arrangement in which the home government has an exogenous expenditure requirement $g \in (0, 1)$: it must consume a fraction g of nominal GDP each period. The government finances this expenditure with seigniorage *and* tariff revenue combined (government budget constraint (27)). The money-growth rate μ is endogenous, but the tariff enters the government budget constraint in a way that leaves the steady-state conditions for n and n^* unaffected. The full formal definitions, equilibrium conditions, and steady-state derivations are in Appendix B.

Proposition 5 (Case A: Tariff Neutrality with Fiscal Spending)**.** *In Case A, the steady-state labor allocations n and n^* are invariant to the tariff rate θ. There exists a unique inflation rate $\mu \in \left(1, \frac{1}{1-g}\right)$ satisfying the government budget constraint, independently of θ.*

Proof. See Appendix A.7. □

After imposing steady state, n and n^* are pinned down solely by μ (as in the benchmark), and the budget constraint determines μ independently of θ.

4.4 Robustness: Multi-Country Environment with Portfolio Choice

The benchmark model assumes that the home transaction asset is the only store of value and that the foreign economy is a single country. Appendix D relaxes both assumptions by developing a multi-country environment in which a continuum of foreign economies produce differentiated goods (Dixit-Stiglitz varieties), trade with each other using the home currency for cross-border settlement, and allocate savings between home money and a competing foreign settlement instrument. The portfolio share between the two currencies is determined endogenously by a micro-founded transaction problem: each cross-border purchase requires currency, and different markets have different acceptance rates for the reserve currency, generating a CES liquidity aggregator from primitives.

The central result is that tariff neutrality survives in this richer environment (Proposition 9 in Appendix D). The logic is the same as in the benchmark, but now applies to a setting with endogenous portfolio choice. The import tariff θ creates a wedge in goods prices, but the portfolio share s—which determines the net real transfer and thus the trade deficit—depends on asset returns $(1/\mu, 1/\hat{\mu})$ and the structure of international payment networks (λ, σ), not on goods prices. Because the tariff operates on the goods-price margin while the trade deficit is pinned by the asset-return margin, the two are orthogonal: the tariff cannot alter the portfolio share, the foreign labor allocation, or the import quantity.

This orthogonality does not depend on the specific functional form of the portfolio problem. As shown in Appendix D, the neutrality holds for any smooth, homogeneous-of-degree-one liquidity aggregator $L(a, b)$, not just CES. The key requirement is only that the tariff does not enter the asset-return condition that determines the portfolio share—a property that holds whenever the tariff is a pure goods-price wedge and revenue is rebated to domestic agents.

The appendix also contrasts the goods tariff with a *financial tariff* τ_f on foreign reserve holdings. Because the financial tariff reduces the after-tax return on the home asset, it operates directly on the asset-return margin: it shifts the portfolio share, reduces the trade deficit, and expands home tradables employment. The contrast between the neutral goods tariff and the effective financial tariff sharpens the paper's main message: in a reserve-currency economy, the relevant instrument for external rebalancing is financial policy, not trade policy.

5 Conclusion

If the U.S. trade deficit is the equilibrium counterpart of foreign demand for dollar-denominated assets, then the deficit cannot be eliminated by making imports more expensive. The foreign economy exports tradables not because it wants to, but because it needs dollars. A tariff does not change that need.

We have built a model to make this argument precise. In equilibrium, the foreign labor allocation and the import quantity are governed entirely by the foreign economy's demand for the home transaction asset—both are independent of the tariff rate. At home, the tariff raises import prices, but when revenue is rebated to consumers their spending power is restored. The home labor market sees no change in the relative return to tradables production. Sectoral employment and output are invariant.

What does change is relative prices. A higher tariff compresses the foreign producer price relative to the home consumer price, reducing the measured trade-deficit-to-GDP ratio through valuation rather than through quantity adjustment. The burden is shared: foreign exporters receive less per unit, home consumers pay more per unit, and the home government collects the difference.

Several extensions refine this message. Imperfect substitutability between home and foreign tradables in the one-way CES extension does not restore tariff effectiveness: long-run quantities remain invariant, and only the valuation channel operates (Proposition 3). In the symmetric Armington appendix, tariffs can compress gross trade flows and may further reduce home tradables employment, but they still do not undo the reserve-demand mechanism governing the net transfer. A production subsidy to home tradables does raise steady-state employment in the goods sector, because it operates directly on the margin that tariffs miss: the relative return to producing tradables (Proposition 4). The fiscal disposition of tariff revenue matters: if revenue finances a fixed government expenditure rather than being rebated to consumers, neutrality continues to hold (Proposition 5). Most importantly, the neutrality result is not an artifact of the benchmark's single-currency assumption: it survives in a multi-country environment with differentiated goods, a foreign settlement currency, and endogenous portfolio choice (Proposition 9). In that richer environment, a financial tariff on foreign reserve holdings—unlike a goods tariff—does reduce the trade deficit and expand home tradables employment, because it operates on the asset-return margin rather than the goods-price margin (Proposition 10).

The broader lesson is about the interaction between trade policy and the monetary system. In a reserve-currency economy, the real trade balance is determined by asset demand, not by trade policy. Tariffs can redistribute income between countries and raise domestic revenue; they cannot rebalance trade or reindustrialize the economy on their own. A policy aimed at reducing the trade deficit in real terms would need to address the underlying demand for U.S.-issued assets—a much harder problem than adjusting the tariff schedule.

Several important dimensions are left for future work. The model abstracts from aggregate demand, nominal rigidities, and short-run exchange-rate dynamics, all of which may give tariffs non-trivial transitional effects even when long-run allocations are neutral. We also set aside welfare analysis: while higher tariffs raise home fiscal revenue and shift the terms of trade in some scenarios, the optimal tariff problem in a reserve-currency setting—balancing these gains against the costs of higher domestic consumer prices and the risk of eroding reserve-currency status—remains an open question. Finally, introducing a discrete erosion of reserve-currency status as a response to high tariffs would add a strategic dimension not captured here. These extensions are left for future research.

A Proofs

A.1 Proof of Lemma 1

Start from the demand for service in foreign country

$$p_t^{c*} \sum_{i=1,2,3} c_t^{i*} = (1-\phi) \sum_{i=1,2,3} w_{t-1}^{i*}$$

On the left-hand side, total consumption of service equals total production. On the right-hand side, total income in the foreign economy equals total value of output. Therefore,

$$p_t^{c*} F(1-n_t^*) = (1-\phi) \left(p_{t-1}^k n_{t-1}^{*\alpha} + p_{t-1}^{c*} \left(1-n_{t-1}^*\right)^{\alpha} \right)$$

The optimal choice by the type 3 worker implies

$$\frac{p_t^k}{p_t^{c*}} = \frac{n_t^*}{1-n_t^*}\frac{(1-n_t^*)^\alpha}{n_t^{*\alpha}}$$

Using this to simplify gives equation (6):

$$(1-n_t^*)\, n_t^{*\alpha-1} = (1-\phi)\,\frac{p_{t-1}^k}{p_t^k} n_{t-1}^{*\alpha-1}$$

Similarly, demand for goods in the foreign economy is

$$\sum_{i=1,2,3} k_t^{i*} = \phi \frac{p_{t-1}^k}{p_t^k} n_{t-1}^{*\alpha-1}$$

The demand for service in the home economy is

$$p_t^c \sum_{i=1,2,3} c_t^i = (1-\phi)(\sum_{i=1,2,3} w_{t-1}^i + (\mu-1)M_{t-1})$$

where

$$\sum_{i=1,2,3} w_{t-1}^i = p_{t-1}^k n_{t-1}^\alpha + p_{t-1}^c\,(1-n_{t-1})^\alpha$$

and

$$M_{t-1} = p_{t-1}^k\left(n_{t-1}^\alpha + n_{t-1}^{*\alpha}\right) + p_{t-1}^{c*}\left(1-n_{t-1}^*\right)^\alpha + p_{t-1}^c\,(1-n_{t-1})^\alpha$$

The first equation is the national income accounting identity. The second is the total demand for money by the young generation of cohort $t-1$. The optimal sector choice of type 3 at home implies

$$\frac{p_t^k}{p_t^c} = \frac{n_t}{1-n_t}\frac{(1-n_t)^\alpha}{n_t^\alpha}$$

Using these equations to simplify service demand at home gives equation (7):

$$(1-n_t)\, n_t^{\alpha-1} = (1-\phi)\,\frac{p_{t-1}^k}{p_t^k}\left(\mu n_{t-1}^{\alpha-1} + (\mu-1)\, n_{t-1}^{*\alpha-1}\right)$$

Similarly, demand for goods in the home economy is

$$k_t = \phi \frac{p_{t-1}^k}{p_t^k}\left(\mu n_{t-1}^{\alpha-1} + (\mu-1)\, n_{t-1}^{*\alpha-1}\right)$$

Adding demand for goods in home and foreign economies and using goods-market clearing gives equation (8):

$$n_t^{\alpha} + n_t^{*\alpha} = \phi\mu \frac{p_{t-1}^k}{p_t^k} \left(n_{t-1}^{\alpha-1} + n_{t-1}^{*\alpha-1}\right)$$

The initial old in the home economy hold the initial stock of money M_0 and can buy goods and services. Service demand at home together with service-market clearing implies equation (10):

$$p_0^c \left(1 - n_0\right)^{\alpha} = \left(1 - \phi\right) M_0.$$

The initial old in the foreign economy have no money and hence no consumption. Since foreign services are non-tradable and no one can pay for them, no services are produced. All labor goes to goods production, which is shipped to the home economy in exchange for money. Together with optimal goods demand in the domestic economy this implies equation (11):

$$p_0^k \left(n_0^{*\alpha} + n_0^{\alpha}\right) = \phi M_0.$$

Finally, equation (12) is the optimal sector choice for type 3 households in the home economy.

A.2 Proof of Proposition 1

Imposing a steady-state condition in (6) we get

$$1 - n^* = \frac{1-\phi}{\mu}$$

and therefore

$$n^* = 1 - \frac{1-\phi}{\mu}.$$

Therefore, $n^* \geq \phi$ with strict inequality if $\mu > 1$.

Next, imposing the steady-state condition in (7):

$$\frac{1-n}{n} n^{\alpha} = (1-\phi)\left(\frac{n^{\alpha}}{n} + \frac{\mu-1}{\mu}\frac{n^{*\alpha}}{n^*}\right)$$

Therefore, $n \leq \phi$ (since $\mu \geq 1$) with a strict inequality if $\mu > 1$. This establishes the first claim.

For the second claim, the ratio of the wage of a goods producer to that of a service producer in the home country is

$$\frac{w_t^1}{w_t^2} = \frac{p_t^k}{p_t^c} \frac{n^\alpha}{(1-n)^\alpha}$$
$$= \frac{n}{1-n}$$
$$\leq \frac{\phi}{1-\phi},$$

with equality if and only if $\mu = 1$. A similar argument establishes $\frac{w_t^{1*}}{w_t^{2*}} \geq \frac{\phi}{1-\phi}$. This establishes the second claim.

To establish the last claim, the equilibrium condition gives

$$\frac{\text{EXP}_t}{\text{GDP}_t} = \frac{p_{t-1}^k}{p_t^k} \frac{\mu n_{t-1}^{\alpha-1} + (\mu - 1)\, n_{t-1}^{*\alpha-1}}{n_t^{\alpha-1}}$$

In steady state:

$$\frac{\text{EXP}_t}{\text{GDP}_t} = \frac{1}{\mu} \frac{\mu n^{\alpha-1} + (\mu-1)\, n^{*\alpha-1}}{n^{\alpha-1}}$$
$$= 1 + \frac{\mu - 1}{\mu} \frac{n^{*\alpha}}{n^\alpha} \frac{n}{n^*}$$
$$\geq 1$$

with equality if and only if $\mu = 1$. A similar argument establishes $\text{EXP}^*/\text{GDP}^* \leq 1$.

A.3 Proof of Lemma 2

Let $K_{jt} = \sum_{i=1,2,3} k_{jt}^i$ for $j = H, F$ and $K_{Ft}^* = \sum_{i=1,2,3} k_{Ft}^{i*}$ be aggregate consumption of goods in the domestic and foreign economies, respectively. The allocation in the foreign economy must satisfy the following conditions.

$$p_t^{c*} (1 - n_t^*)^\alpha = (1-\phi) \left(p_{t-1}^{k*} n_{t-1}^{*\alpha} + p_{t-1}^{c*} \left(1 - n_{t-1}^*\right)^\alpha \right)$$
$$p_t^{k*} K_{Ft}^* = \phi \left(p_{t-1}^{k*} n_{t-1}^{*\alpha} + p_{t-1}^{c*} \left(1 - n_{t-1}^*\right)^\alpha \right)$$

From the optimal sectoral choice:

$$\frac{p_t^{c*}}{p_t^{k*}} = \frac{n_t^{*\alpha}}{(1-n_t^*)^\alpha} \frac{(1-n_t^*)}{n_t^*}$$

Using this:

$$n_t^{*\alpha-1} - n_t^{*\alpha} = (1-\phi)\,\frac{p_{t-1}^{k*}}{p_t^{k*}} n_{t-1}^{*\alpha-1}$$
$$K_{Ft}^{*} = \phi \frac{p_{t-1}^{k*}}{p_t^{k*}} n_{t-1}^{*\alpha-1}$$

The first equation is (16), and the second is optimal demand for foreign-produced goods in the foreign economy. Allocation in the home economy must satisfy

$$p_t^c\,(1-n_t)^\alpha = (1-\phi)\left(p_{t-1}^k n_{t-1}^\alpha + p_{t-1}^c\,(1-n_{t-1})^\alpha + (\mu-1)\,M_{t-1} + \theta p_t^{k*} K_{Ft}\right)$$
$$p_t^k K_{Ht} + (1+\theta)\,p_t^{k*} K_{Ft} = \phi\left(p_{t-1}^k n_{t-1}^\alpha + p_{t-1}^c\,(1-n_{t-1})^\alpha + (\mu-1)\,M_{t-1} + \theta p_t^{k*} K_{Ft}\right)$$

Replacing the equilibrium value of M_{t-1} and K_{Ht} and using the optimal sectoral choice condition, these simplify to

$$n_t^{\alpha-1} - n_t^{\alpha} = (1-\phi)\left(\mu \frac{p_{t-1}^k}{p_t^k} n_{t-1}^{\alpha-1} + (\mu-1)\,\frac{p_{t-1}^{k*}}{p_t^k} n_{t-1}^{*\alpha-1} + \theta \frac{p_t^{k*}}{p_t^k} K_{Ft}\right)$$
$$n_t^{\alpha} + (1+\theta)\,\frac{p_t^{k*}}{p_t^k} K_{Ft} = \phi\left(\mu \frac{p_{t-1}^k}{p_t^k} n_{t-1}^{\alpha-1} + (\mu-1)\,\frac{p_{t-1}^{k*}}{p_t^k} n_{t-1}^{*\alpha-1} + \theta \frac{p_t^{k*}}{p_t^k} K_{Ft}\right)$$

The equilibrium imports in the home country are

$$K_{Ft} = n_t^{*\alpha} - K_{Ft}^{*} = n_t^{*\alpha} - \phi \frac{p_{t-1}^{k*}}{p_t^{k*}} n_{t-1}^{*\alpha-1}$$

Substituting for K_{Ft} and $p_t^k = (1+\theta)p_t^{k*}$ gives equations (17) and (18).

A.4 Proof of Proposition 2

Part (i). Impose the steady-state condition $n_t^* = n^*$ in (16):

$$\frac{1-n^*}{n^*} n^{*\alpha} = (1-\phi)\frac{1}{\mu}\frac{n^{*\alpha}}{n^*},$$

which simplifies to $1-n^* = (1-\phi)/\mu$, so $n^* = 1-(1-\phi)/\mu$. This depends only on μ and ϕ; it is independent of θ.

Part (ii). Imposing steady state in (17) and substituting $p_t^k = (1+\theta)p_t^{k*}$, after substituting the expression for n^* and rearranging, the equilibrium condition for n reduces to

$$\phi\frac{n^\alpha}{n} - n^\alpha = (1-\phi)\left(\frac{\mu-1}{\mu}\right)\frac{n^{*\alpha}}{n^*}.$$

The right-hand side depends only on n^* (independent of θ by part (i)), μ, and ϕ. Hence n is independent of θ.

Part (iii). In steady state, foreign old-age goods demand is $k_F^* = (\phi/\mu)n^{*\alpha-1}$. Hence home imports are

$$K_F = n^{*\alpha} - \frac{\phi}{\mu}n^{*\alpha-1} = \frac{\mu-1}{\mu}\,n^{*\alpha-1},$$

which depends only on μ, ϕ, α, and n^*, all independent of θ. □

A.5 Proof of Proposition 3

Foreign allocation. The foreign equilibrium is unchanged relative to the benchmark. Hence $n^* = 1-(1-\phi)/\mu$ and $K_F = [(\mu-1)/\mu]n^{*\alpha-1}$, both independent of θ (cf. Proposition 2, parts (i) and (iii)).

Home labor allocation. From the steady-state home system (21)–(22), define $s \equiv (1+\theta)p^{k*}/p^k$. Equation (22) gives n as a strictly decreasing function of s:

$$n = \frac{(1-\eta)\phi}{(1-\phi)\eta\, s^{1-\rho} + (1-\eta)}.$$

Substituting into (21) gives a single equation in s whose solution is independent of θ because the right-hand side of (21) depends on θ only through s and K_F (both already pinned down). Hence n is independent of θ.

Trade-deficit ratio. The formula in the main text shows that the ratio equals (p^{k*}/p^k) times a quantity term that is fixed in steady state. It therefore falls with θ through the price ratio alone. □

A.6 Proof of Proposition 4

Part (i). The foreign problem is unchanged by κ. Hence $n^* = 1-(1-\phi)/\mu$ and $K_F = [(\mu-1)/\mu]n^{*\alpha-1}$, both independent of κ.

Part (ii). In steady state, the home equilibrium condition is

$$H(n,\kappa) \equiv (1+\kappa)\phi n^{\alpha-1} - (1+\phi\kappa)n^\alpha - (1-\phi)\frac{\mu-1}{\mu}n^{*\alpha-1} = 0.$$

By the implicit function theorem, $dn/d\kappa = -H_\kappa/H_n$. We have

$$H_\kappa = \phi n^{\alpha-1} - \phi n^{\alpha} = \phi n^{\alpha-1}(1-n) > 0,$$

and

$$H_n = (1+\kappa)\phi(\alpha-1)n^{\alpha-2} - (1+\phi\kappa)\alpha n^{\alpha-1} = n^{\alpha-2}\big[(1+\kappa)\phi(\alpha-1) - (1+\phi\kappa)\alpha n\big].$$

For $n \in (0,1)$ and $\alpha \in (0,1)$ both terms in brackets are negative, so $H_n < 0$. Therefore $dn/d\kappa = -H_\kappa/H_n > 0$. □

A.7 Proof of Proposition 5

Imposing steady state in the Case A system (Appendix B, equations (28)–(29)), the labor allocations satisfy $n^* = 1-(1-\phi)/\mu$ and $n = 1-(1-\phi)/[\mu(1-g)]$, exactly as in the benchmark but with endogenous μ. The money-growth rate is determined by the government budget constraint (30):

$$\big(\mu(1-g)-1\big)n^{\alpha-1} + \frac{\mu-1}{\mu}n^{*\alpha-1} = 0.$$

Substituting the expressions for n and n^*, define

$$\tilde{F}(\mu) \equiv \big((1-g)\mu - 1\big)\left(\mu - \frac{1-\phi}{1-g}\right)^{\alpha-1} + (\mu-1+\phi)^{\alpha-1}(\mu-1).$$

The function $\tilde{F}$ is strictly increasing on $\big(\frac{1-\phi}{1-g}, \frac{1}{1-g}\big)$, with $\tilde{F} \to -\infty$ at the left endpoint and $\tilde{F} > 0$ at $\mu = \frac{1}{1-g}$. By the intermediate value theorem there is a unique $\hat{\mu} \in \big(\frac{1-\phi}{1-g}, \frac{1}{1-g}\big)$. One further verifies $\hat{\mu} > 1$. Since θ does not appear in $\tilde{F}$, $\hat{\mu}$, n, and n^* are all independent of θ. □

B Alternative Fiscal Policy

B.1 Closed Economy

Consider the same economy as in the main text. The domestic government has exogenously given expenditures and consumes a constant fraction g of the output of goods and services. This expenditure is financed by issuing nominal asset M_t. Therefore, the domestic government's budget constraint is

$$M_t - M_{t-1} = g\left(p_t^c\,(1-n_t)^\alpha + p_t^k n_t^\alpha\right) \tag{26}$$

Here, g is the exogenous policy choice, and the path of M_t that finances the expenditure is an endogenous equilibrium object.

Definition. *A closed economy competitive equilibrium is a sequence of worker consumption allocations and money holdings* $\{c_t^i, k_t^i, m_t^i\}_{t=0, i\in\{1,2,3\}}^{\infty}$*, the fraction of type 3 households in the goods sector* $\{n_t\}_{t=0}^{\infty}$*, prices* $\{p_t^c, p_t^k\}_{t=0}^{\infty}$*, wages* $\{w_t^i\}_{t=0, i\in\{1,2,3\}}^{\infty}$*, and government policy* $\{M_t, g\}_{t=0}^{\infty}$*, such that*

1. Given prices, wages and government policy, allocation $\{c_t^i, k_t^i\}_{t=0}^{\infty}$ solves the problem of individual worker i'
$$\max u\left(c_{t+1}^i, k_{t+1}^i\right)$$
s.t.
$$m_t^i \leq w_t^i$$
$$p_{t+1}^c c_{t+1}^i + p_{t+1}^k k_{t+1}^i \leq m_t^i$$

2. The wages are determined according to equations (1), (2) and (3).

3. The government budget constraint (26) holds.

4. Markets clear
$$\sum_{i=1,2,3} c_t^i = (1-g)\left(1-n_t\right)^{\alpha}$$
$$\sum_{i=1,2,3} k_t^i = (1-g)\, n_t^{\alpha}$$
$$\sum_{i=1,2,3} m_t^i = M_t$$

Optimal demand by old households is given by

$$p_{t+1}^k (1-g)\, n_{t+1}^{\alpha} = \phi M_t$$
$$p_{t+1}^c (1-g)\left(1-n_{t+1}\right)^{\alpha} = (1-\phi)\, M_t$$

This implies
$$\frac{p_{t+1}^k}{p_{t+1}^c}\frac{n_{t+1}^{\alpha}}{\left(1-n_{t+1}\right)^{\alpha}} = \frac{\phi}{1-\phi}.$$

Optimal sector choice by young mobile households implies

$$\frac{p_{t+1}^k}{p_{t+1}^c} = \frac{(1-n_{t+1})^{\alpha-1}}{n_{t+1}^{\alpha-1}}.$$

Combining these equations yields

$$n_{t+1} = \phi.$$

Total income of young households in period t equals total demand for assets:

$$M_t = p_t^k n_t^\alpha + p_t^c (1-n_t)^\alpha = p_t^k n_t^{\alpha-1},$$

where the second equality follows from optimal sector choice. Total expenditures in period $t+1$ equal the asset position of old households. Aggregate private consumption of goods and services in period $t+1$ is $(1-g)\, n_t^\alpha$ and $(1-g)(1-n_t)^\alpha$, respectively. Therefore,

$$M_t = p_{t+1}^k (1-g)\, n_{t+1}^\alpha + p_{t+1}^c (1-g)(1-n_{t+1})^\alpha = p_{t+1}^k n_{t+1}^{\alpha-1}$$

Combining these equations and replacing $n_t = \phi$ gives

$$\frac{p_{t+1}^k}{p_t^k} = \frac{1}{1-g}.$$

and therefore

$$\mu \equiv \frac{M_{t+1}}{M_t} = \frac{1}{1-g}.$$

In the closed economy, money grows at a constant rate $\mu = \frac{1}{1-g}$, resulting in a constant inflation rate $\frac{p_{t+1}^c}{p_t^c} = \frac{p_{t+1}^k}{p_t^k} = \frac{1}{1-g}$.

B.2 Trade: Tariff is used to finance deficit

We now introduce a foreign country. As in the main text, only the domestic economy can issue money. The foreign economy must export goods to the domestic economy to acquire money; otherwise no production takes place in the foreign economy.

The home government levies a tariff at rate θ on imported goods. Revenue from tariffs is used to finance expenditures.

Definition. *A tariff distorted competitive equilibrium is a sequence of domestic and foreign allocations and money holdings* $\{(c_t^i, k_{Ht}^i, k_{Ft}^i, m_t^i), (c_t^{i*}, k_{Ft}^{i*}, m_t^{i*})\}_{t=0,\, i\in\{1,2,3\}}^\infty$*, mobile-labor allocations* $\{n_t, n_t^*\}_{t=0}^\infty$*, prices* $\{p_t^c, p_t^{c*}, p_t^k, p_t^{k*}\}_{t=0}^\infty$*, wages* $\{w_t^i, w_t^{i*}\}_{t=0,\, i\in\{1,2,3\}}^\infty$*, and government policy* $\{\theta, M_t, g\}_{t=0}^\infty$*, such that*

1. *Given prices, wages and government policy, allocation* $\{c_t^i, k_t^i, m_t^i\}_{t=0}^{\infty}$ *solves the problem of domestic individual worker* i*'*

$$\max u\left(c_{t+1}^i, k_{Ht+1}^i + k_{Ft+1}^i\right)$$

s.t.

$$m_t^i \leq w_t^i$$
$$p_{t+1}^c c_{t+1}^i + p_{t+1}^k k_{Ht+1}^i + (1+\theta)\, p_{t+1}^{k*} k_{Ft+1}^i \leq m_t^i$$

2. *Given prices, wages, and government policy, allocation* $\{c_t^{i*}, k_t^{i*}, m_t^{i*}\}_{t=0}^{\infty}$ *solves the problem of the foreign individual worker* i*' (note: the difference is that they do not receive the transfer)*

$$\max u\left(c_{t+1}^{i*}, k_{Ft+1}^{i*}\right)$$

s.t.

$$m_t^{i*} \leq w_t^{i*}$$
$$p_{t+1}^{c*} c_{t+1}^{i*} + p_{t+1}^{k*} k_{Ft+1}^{i*} \leq m_t^{i*}$$

3. *Domestic wages are determined according to equations (1), (2) and (3) (and similarly for foreign wages).*

4. *The government budget constraint holds*

$$M_t - M_{t-1} + \theta p_t^{k*} \left(\sum_{i=1,2,3} k_{Ft}^i \right) = g\left(p_t^c\left(1-n_t\right)^{\alpha} + p_t^k n_t^{\alpha}\right) \tag{27}$$

5. *Allocation is feasible*

$$\sum_{i=1,2,3} c_t^i = (1-g)(1-n_t)^\alpha$$
$$\sum_{i=1,2,3} c_t^{i*} = (1-n_t^*)^\alpha$$
$$\sum_{i=1,2,3} k_{Ht}^i = (1-g)\, n_t^\alpha$$
$$\sum_{i=1,2,3} k_{Ft}^i + \sum_{i=1,2,3} k_{Ft}^{i*} = n_t^{*\alpha}$$
$$\sum_{i=1,2,3} m_t^i + \sum_{i=1,2,3} m_t^{i*} = M_t$$

The law of one price must hold in equilibrium: $p_t^k = (1+\theta)\, p_t^{k*}$. Imposing market clearing and households' optimal choices of service consumption:

$$p_{t+1}^{c*}\left(1-n_{t+1}^*\right)^\alpha = (1-\phi)\left(p_t^{k*}(n_t^*)^\alpha + p_t^{c*}(1-n_t^*)^\alpha\right) \tag{28}$$
$$(1-g)\, p_{t+1}^{c}(1-n_{t+1})^\alpha = (1-\phi)\left(p_t^k n_t^\alpha + p_t^c (1-n_t)^\alpha\right) \tag{29}$$

Imposing steady state and using optimal sector choice equations (19) and (20):

$$\mu(1-n^*)^\alpha = (1-\phi)(1-n^*)^{\alpha-1}$$
$$(1-g)\,\mu(1-n)^\alpha = (1-\phi)(1-n)^{\alpha-1}$$

Therefore,

$$n^* = 1 - \frac{1-\phi}{\mu}$$
$$n = 1 - \frac{1-\phi}{\mu(1-g)}$$

The labor allocation in the goods sector is always higher in the foreign economy.

To solve for the unknown growth rate μ, the aggregate demand for goods in the foreign economy is

$$p_{t+1}^{k*} k_{Ft+1}^* = \phi\left(p_t^{k*}(n_t^*)^\alpha + p_t^{c*}(1-n_t^*)^\alpha\right)$$

which simplifies to

$$k_{Ft+1}^* = \frac{\phi}{\mu}(n_t^*)^{\alpha-1}.$$

Home imports in steady state are

$$(n^*)^\alpha - k_F^* = \left(\frac{\mu - 1}{\mu}\right)(n^*)^{\alpha-1}$$

The total demand for assets by home and foreign young households must equal the supply:

$$\begin{aligned} M_t &= p_t^k n_t^\alpha + p_t^c (1 - n_t)^\alpha + p_t^{k*} (n_t^*)^\alpha + p_t^{c*} (1 - n_t^*)^\alpha \\ &= p_t^k \left(n_t^{\alpha-1} + \frac{(n_t^*)^{\alpha-1}}{1+\theta}\right) \end{aligned}$$

Substituting into the government budget constraint:

$$\begin{aligned} p_t^k \left(n_t^{\alpha-1} + \frac{(n_t^*)^{\alpha-1}}{1+\theta}\right) - p_{t-1}^k \left(n_{t-1}^{\alpha-1} + \frac{\left(n_{t-1}^*\right)^{\alpha-1}}{1+\theta}\right) \\ = p_t^k \left(g n_t^{\alpha-1} + \frac{\theta}{1+\theta}\left((n_t^*)^\alpha - \frac{\phi}{\mu}\left(n_{t-1}^*\right)^{\alpha-1}\right)\right) \end{aligned}$$

Imposing steady state and rearranging:

$$\left(1 - g - \frac{1}{\mu}\right) n^{\alpha-1} + \left(\frac{\mu-1}{\mu}\right)(n^*)^{\alpha-1} = 0 \tag{30}$$

where $n^* = 1 - \frac{1-\phi}{\mu}$ and $n = 1 - \frac{1-\phi}{\mu(1-g)}$.

The tariff has no long-term impact on allocation.

Proposition 6. *In equilibrium, there is a unique inflation rate μ such that $1 < \mu < \frac{1}{1-g}$.*

Proof. Define

$$F(\mu) = ((1-g)\mu - 1)\left(\mu - \frac{1-\phi}{1-g}\right)^{\alpha-1} + (\mu - 1 + \phi)^{\alpha-1}(\mu - 1)$$

Step 1. The equation above has a unique solution in the interval $\left(\frac{1-\phi}{1-g}, \frac{1}{1-g}\right)$.

$F(\cdot)$ is monotonically increasing on the interval $\left(\frac{1-\phi}{1-g}, \frac{1}{1-g}\right)$:

$$\begin{aligned}
F'(\mu) &= (1-g)\left(\mu - \frac{1-\phi}{1-g}\right)^{\alpha-1} + (\alpha-1)\left((1-g)\mu - 1\right)\left(\mu - \frac{1-\phi}{1-g}\right)^{\alpha-2} \\
&+ (\mu - 1 + \phi)^{\alpha-1} + (\alpha-1)(\mu-1+\phi)^{\alpha-2}(\mu-1) \\
&= \left(\mu - \frac{1-\phi}{1-g}\right)^{\alpha-2}\left((1-g)\left(\mu - \frac{1-\phi}{1-g}\right) + (\alpha-1)\left((1-g)\mu-1\right)\right) \\
&+ (\mu-1+\phi)\left((\mu-1+\phi) + (\alpha-1)(\mu-1)\right) \\
&= \left(\mu - \frac{1-\phi}{1-g}\right)^{\alpha-2}\left((1-g)\mu - (1-\phi) + (\alpha-1)\left((1-g)\mu-1\right)\right) \\
&+ (\mu-1+\phi)^{\alpha-2}\left(\mu - (1-\phi) + (\alpha-1)(\mu-1)\right) \\
&> 0
\end{aligned}$$

where the last inequality holds because both terms are positive for $\mu \in \left(\frac{1-\phi}{1-g}, \frac{1}{1-g}\right)$.

Evaluating F on the boundary:

$$F\left(\frac{1-\phi}{1-g}\right) = -\phi\left(\frac{1-\phi}{1-g} - \frac{1-\phi}{1-g}\right)^{\alpha-1} + \left(\frac{1-\phi}{1-g} - 1 + \phi\right)^{\alpha-1}\left(\frac{1-\phi}{1-g} - 1\right) = -\infty$$

and

$$F\left(\frac{1}{1-g}\right) = \left(\frac{1}{1-g} - 1 + \phi\right)^{\alpha-1}\left(\frac{1}{1-g} - 1\right) > 0$$

By the intermediate value theorem, there is a unique $\hat{\mu}$ such that $F(\hat{\mu}) = 0$.

Step 2. $\hat{\mu} > 1$.

In the equation

$$\left((1-g)\mu - 1\right)\left(\mu - \frac{1-\phi}{1-g}\right)^{\alpha-1} + (\mu-1+\phi)^{\alpha-1}(\mu-1) = 0$$

we know $\hat{\mu} < \frac{1}{1-g}$, so $\left((1-g)\hat{\mu} - 1\right)\left(\hat{\mu} - \frac{1-\phi}{1-g}\right)^{\alpha-1} < 0$. If $\hat{\mu} \leq 1$, then $F(\hat{\mu}) < 0$, contradicting the definition of $\hat{\mu}$. Therefore,

$$1 < \hat{\mu} < \frac{1}{1-g}.$$

□

C Symmetric Armington: Two-Way Trade

This appendix develops the symmetric Armington extension summarized in Section 4. Both countries have CES demand over home and foreign tradable varieties. Let $q \equiv p^k/p^{k*}$ denote the relative producer price of the home variety. Home households have Armington elasticity $\rho > 1$ and home-variety weight $\eta \in (0,1)$; foreign households have elasticity $\gamma > 1$ and foreign-variety weight $1-\eta^*$ for $\eta^* \in (0,1)$. The home country levies an ad valorem tariff $\theta \geq 0$ on the foreign variety, so the tariff-inclusive home import price is $(1+\theta)p^{k*}$.

Lemma 3 (Armington import ratios)**.** *In an interior allocation, Armington demand implies*

$$\begin{aligned} \frac{k_F}{k_H} &= \frac{1-\eta}{\eta}\left(\frac{1+\theta}{q}\right)^{-\rho}, \\ \frac{k_H^*}{k_F^*} &= \frac{\eta^*}{1-\eta^*}\, q^{-\gamma}. \end{aligned} \tag{31}$$

Proof. Standard CES cost minimization for the tradables aggregator in each country, accounting for the tariff-inclusive price $(1+\theta)p^{k*}$ facing home consumers. □

Lemma 4 (Gross flows given (q,θ))**.** *Let $a(q,\theta) \equiv \frac{1-\eta}{\eta}(1+\theta)^{-\rho}q^{\rho}$ and $b(q) \equiv \frac{\eta^*}{1-\eta^*}q^{-\gamma}$. Market clearing for each variety yields*

$$\begin{aligned} k_F(q,\theta) &= \frac{a(q,\theta)\big(n^{\alpha} - b(q)n^{*\alpha}\big)}{1-a(q,\theta)b(q)}, \\ k_H^*(q,\theta) &= b(q)\big(n^{*\alpha} - k_F(q,\theta)\big). \end{aligned}$$

Holding q fixed, k_F is strictly decreasing in θ. Under $ab < 1$, k_H^* is strictly decreasing in q.

Proof. Substitute (31) into the market-clearing condition for the home variety and solve the resulting 2×2 system; monotonicity follows from $\partial a/\partial\theta < 0$ and $\partial b/\partial q < 0$. □

In steady state, the reserve-currency mechanism pins down $n^* = 1-(1-\phi)/\mu$ and therefore the net real resource transfer $T = (1-1/\mu)n^{*\alpha-1}$. The balance-of-payments identity requires

$$k_F - q\,k_H^* = T,$$

which is independent of θ. Define $\mathcal{F}(q,\theta) \equiv k_F(q,\theta) - q\,k_H^*(q,\theta) - T$; an equilibrium relative price $q(\theta)$ solves $\mathcal{F}(q,\theta) = 0$.

Proposition 7 (Terms-of-trade appreciation)**.** *If $\mathcal{F}_q > 0$ at an equilibrium (sufficient condition: $ab < 1$ and $\rho > \gamma$), then $dq/d\theta > 0$.*

Proof. By the IFT, $dq/d\theta = -\mathcal{F}_\theta/\mathcal{F}_q$. From Lemma 4, $\mathcal{F}_\theta = \partial k_F/\partial\theta < 0$ at fixed q. When $\rho > \gamma$ and $ab < 1$, the home substitution channel dominates the foreign channel, giving $\mathcal{F}_q > 0$, and hence $dq/d\theta > 0$. □

Proposition 8. *If the conditions of Proposition 7 hold and the incomplete-offset condition*

$$\frac{d}{d\theta}\ln\left(\frac{1+\theta}{q}\right) > 0,$$

both $dk_F/d\theta < 0$ and $dk_H^*/d\theta < 0$.

Proof. The effective relative import price $(1+\theta)/q(\theta)$ rises by assumption, reducing home import demand ($\rho > 1$). The home relative producer price q rises, reducing foreign demand for the home variety ($\gamma > 1$). □

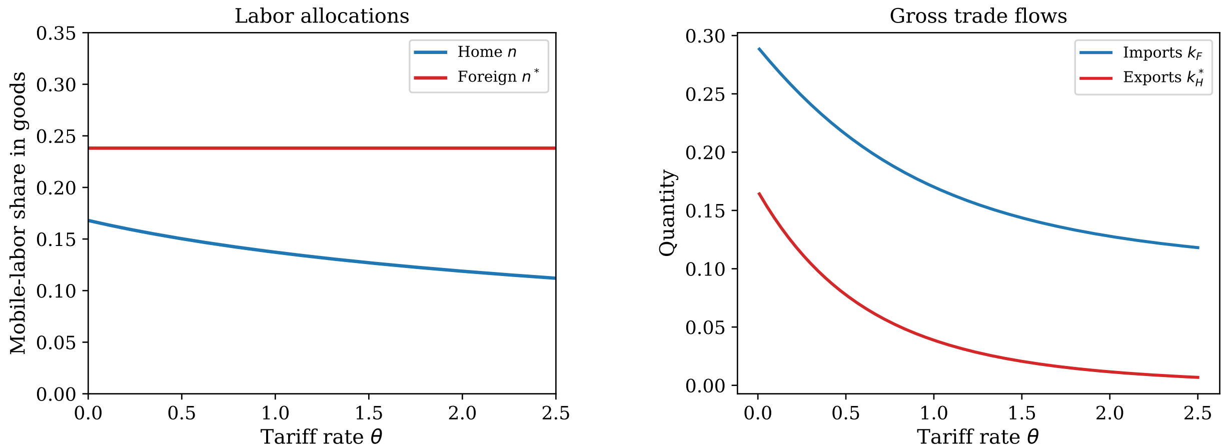
Home labor share. Under the incomplete-offset condition, the Armington system implies that the tariff raises the effective relative import price faced by home consumers. This shifts home expenditure toward services and lowers the equilibrium relative producer price p^k/p^c. Since the home labor-supply schedule is $p^k/p^c = [(1-n)/n]^{\alpha-1}$, a lower relative price of tradables implies a lower home tradables labor share n. Figure 6 illustrates this comparative statics result numerically.

Figure 6 reports a numerical comparative statics exercise. As the tariff increases: (i) the foreign goods-sector labor share is essentially unchanged; (ii) the home goods-sector labor share declines (further deindustrialization); and (iii) both gross trade flows fall.

D Multi-Country Environment with Portfolio Choice

This appendix extends the benchmark model by relaxing the assumption that the home transaction asset is the only store of value and that the foreign economy is a single country. We develop a multi-country environment with differentiated goods, a foreign settlement currency, and a microfounded portfolio problem. Tariff neutrality survives: the import tariff θ does not appear in any steady-state allocation. What *does* affect allocations is financial policy—a tax on foreign reserve holdings—which we contrast with the goods tariff.

Notation. We maintain the notation of the main paper throughout. In particular: ϕ is the Cobb-Douglas expenditure share on goods, α is the production-function exponent, $\mu \geq 1$ is the dominant-asset growth rate, n and n^* are the mobile labor shares in the goods sector at home



(a) Labor allocations.

(b) Gross trade flows.

Figure 6: Armington comparative statics ($\phi = 0.2$, $\alpha = 0.5$, $\mu = 1.05$, $\rho = \gamma = 4$, $\eta = \eta^* = 0.6$). As the import tariff θ increases, the home goods-sector labor share n declines (further deindustrialization, left panel), while the foreign goods-sector labor share n^* remains unchanged. Both gross trade flows—imports k_F and exports k_H^*—fall monotonically with θ (right panel).

and abroad, and $\theta \geq 0$ is the ad-valorem import tariff. To avoid confusion between the two currencies in this extension, we refer to the home currency as the *dominant asset* throughout this appendix.

D.1 Countries and demographics

There is one *home* country and a unit mass of *foreign* countries indexed by $i \in [0, 1]$. All countries share the same OLG structure: agents live for two periods, the young work, the old consume.

Foreign countries. Each foreign country i has three types of workers: type 1 specific to goods variety i, type 2 specific to services, type 3 mobile across the two sectors. There is one unit of each type.

Home country. The home country has a richer labor structure reflecting its ability to produce all varieties. There is a *continuum* of type 1 workers indexed by $i \in [0, 1]$, one for each goods variety i. Each type-1_i worker is specific to variety i. There is one unit of type 2 workers (specific to services) and one unit of type 3 workers (mobile). The mobile workers choose which variety to produce, allocating themselves across the continuum of goods industries and the service sector.

D.2 Goods, production, and preferences

Differentiated goods. There is a unit mass of tradable good varieties indexed by $i \in [0, 1]$. Foreign country i can produce only variety i. The home country can produce *all* varieties $i \in [0, 1]$. The asymmetry between home and foreign is therefore in production capacity (home is diversified, foreign is specialized) and in the financial system (home issues the reserve asset), not in preferences.

Services are non-tradable and homogeneous within each country.

Production. In each foreign country i, variety i is produced using the type-1 worker and any mobile labor allocated to goods: output is $n^{*\alpha}$, where n^* is the mobile labor in goods. Service output is $(1 - n^*)^\alpha$.

At home, each variety i is produced using type-1_i (the fixed factor for variety i) and a measure ν_i of mobile labor: output of variety i is ν_i^α. Mobile workers are freely mobile across all goods industries and services, so in equilibrium they earn the same wage everywhere. Service output is $(1 - n)^\alpha$, where $n = \int_0^1 \nu_i \, di$ is total mobile labor in goods.

Preferences. All agents—home and foreign—have identical preferences. Old agents consume services and a symmetric Dixit-Stiglitz aggregate over goods varieties:

$$K = \left[\int_0^1 k_i^{(\varepsilon-1)/\varepsilon} \, di \right]^{\varepsilon/(\varepsilon-1)}, \tag{32}$$

where k_i is consumption of variety i and $\varepsilon > 1$ is the elasticity of substitution. The overall utility is Cobb-Douglas:

$$U = K^\phi \, c^{1-\phi}.$$

Trade pattern. Since each foreign country produces only one variety but consumes all, foreign countries must trade with each other and with home. The home country produces all varieties, so it *can* self-supply goods—but in equilibrium it also imports foreign varieties (since foreign varieties are perfect substitutes for domestically produced ones within each variety, the law of one price holds variety by variety). The home country's net import position reflects the exorbitant privilege mechanism, not a production disadvantage.

D.3 Currencies

Dominant asset. The home government issues the dominant asset (fiat money) $M_t = \mu M_{t-1}$, $\mu \geq 1$, with seigniorage $\tau_t = (1 - 1/\mu) M_t$ distributed lump-sum to home old agents.

Foreign settlement instrument. Foreign countries collectively maintain a settlement system. It can be interpreted as a composite of bilateral credit arrangements, a secondary vehicle currency (for example, a regional anchor currency), or a multilateral clearing mechanism. We represent it as a single "foreign money" $\hat{M}_t = \hat{\mu}\,\hat{M}_{t-1}$, with $\hat{\mu} \geq 1$ and seigniorage distributed to foreign old agents. The exchange rate is e_t (units of the dominant asset per unit of foreign money).

Home agents hold only the dominant asset (they face no transaction disadvantage domestically and receive seigniorage). Foreign agents allocate their savings between the dominant asset and foreign money.

D.4 Transaction structure and the portfolio problem

We now connect the variety structure to the multi-market shopping problem of the benchmark.

Markets as varieties. A foreign old agent must purchase a continuum of goods varieties and services. Each purchase is a *transaction* that requires currency. The key friction is that different transactions are embedded in different payment networks:

- **Goods variety** $j \neq i$ (cross-border): Country i must buy variety j from country j (or from home). This is a cross-border transaction. A fraction of such transactions—commodity trades, large-volume invoicing, contracts with distant partners—are settled in the reserve currency. The remainder—regional trade, familiar bilateral partners—can be settled through the foreign system.
- **Services** (domestic): Purchased from local providers, settled entirely in local currency.

Dominant-asset acceptance across markets. Index all markets (varieties plus services) by $j \in [0,1]$. Each market j has a dominant-asset acceptance rate $\omega(j) \in [0,1]$: a fraction $\omega(j)$ of sellers accept the dominant asset at face value, while $1-\omega(j)$ accept only foreign money.

Since each foreign country is measure zero, its own-variety demand is negligible. Essentially *all* of a foreign agent's goods spending (fraction ϕ) involves cross-border trade. Within this cross-border trade, the distribution of $\omega(j)$ reflects the settlement structure:

- Markets for commodities, standardized intermediates, and trade with distant partners have $\omega(j)$ near 1 (reserve currency required).
- Markets for regional goods, familiar bilateral trade have $\omega(j)$ near 0 (local settlement system suffices).
- Service markets have $\omega(j) = 0$ (purely domestic).

Effective purchasing power in each market. The agent allocates dominant-asset balances a_j and foreign-money balances b_j to each market j. Her effective purchasing power in market j is:

$$\ell_j = \omega(j)\, R^H a_j + [1 - \omega(j)]\, R^F b_j\,,$$

where $R^H = 1/\mu$ and $R^F = 1/\hat{\mu}$ are the real returns on the dominant asset and foreign money, respectively.

In each market j, the agent converts raw balances into effective purchasing power $v(\ell_j)$, where v is a concave transaction technology ($v' > 0$, $v'' < 0$, $v(0) = 0$). The concavity reflects frictions in each payment channel: congestion, search costs, or conversion inefficiencies that make the marginal unit of liquidity directed at a given market less effective than the first.

Total effective purchasing power is:

$$L = \int_0^1 v(\ell_j)\, dj\,.$$

This is the agent's *budget*, not her utility. She then allocates L across the goods aggregate and services: $\max K^{\phi} c^{1-\phi}$ subject to $P^K K + p^{c*} c = L$, where P^K is the Dixit-Stiglitz price index. Because preferences are linear in L, the young agent's portfolio problem reduces to maximizing L subject to $a + b = w^*$.

Optimal currency allocation. The agent allocates total dominant-asset balances $a = \int a_j\, dj$ and foreign-money balances $b = \int b_j\, dj$ across markets. By concavity of v, each market is optimally served by the currency with the higher effective return. Market j is settled in the dominant asset if $\omega(j)\, R^H > [1 - \omega(j)]\, R^F$, i.e., if

$$\omega(j) > \frac{R^F}{R^H + R^F} = \frac{\mu}{\mu + \hat{\mu}} \equiv \bar{\omega}\,.$$

From the distribution of markets to CES. Let $G(\omega)$ denote the CDF of dominant-asset acceptance rates across markets, with density $g(\omega)$. The dominant-asset portfolio share is $s = S(\bar{\omega}; G)$, a decreasing function of $\bar{\omega}$. Under the isoelastic specification $v(\ell) = \ell^{(\sigma-1)/\sigma}/[(\sigma-1)/\sigma]$ and a distribution $g(\omega)$ that places mass $\lambda > 1/2$ on high-ω markets (reflecting the reserve currency's dominance in international settlement), the aggregate liquidity function takes the CES form:

$$L(R^H a,\, R^F b) = \left[\lambda\, (R^H a)^{\frac{\sigma-1}{\sigma}} + (1-\lambda)\, (R^F b)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{33}$$

The parameter λ inherits its meaning from the trade structure: it is the effective mass of cross-border transactions for which the reserve currency has a comparative advantage. The parameter

σ reflects the dispersion of settlement requirements across markets—high σ means many transactions are near-indifferent between currencies (easy substitution), low σ means the two settlement systems serve sharply distinct roles.

Remark. The results that follow hold for any smooth, homogeneous-of-degree-one aggregator $L(a, b)$ with $L_a, L_b > 0$ and strict quasi-concavity, not just CES. We use CES for closed-form expressions.

D.5 Home country

The home country can produce all varieties and services. Its preferences are the same as any foreign country. The asymmetries are: (i) home issues the reserve currency, and (ii) home has a type-1 worker for every variety.

Young agents: labor allocation. Let p_i^k denote the price of variety i. Each type-1_i worker is the fixed factor in variety i and earns the residual after paying mobile labor. Mobile workers choose which industry to work in. Free mobility of type 3 across all goods industries and services requires a common mobile wage:

$$w^3 = p_i^k \, \alpha \, \nu_i^{\alpha-1} = p^c \, \alpha \, (1-n)^{\alpha-1} \quad \text{for all } i \in [0,1].$$

This pins down the allocation of mobile labor across varieties. Since $\nu_i = (p_i^k / p_j^k)^{1/(1-\alpha)} \nu_j$ for any two varieties, symmetric prices ($p_i^k = p^k$ for all i) imply $\nu_i = n$ for all i: mobile labor is spread uniformly across varieties. In this case, home output of each variety is n^α and total home goods output is $\int_0^1 n^\alpha \, di = n^\alpha$.

Wages are then:

$$\begin{aligned} w_i^1 &= p^k (1-\alpha) \, n^\alpha \quad \text{for each } i \in [0,1], \\ w^2 &= p^c (1-\alpha) \, (1-n)^\alpha, \\ w^3 &= p^k \alpha \, n^{\alpha-1} = p^c \alpha \, (1-n)^{\alpha-1}. \end{aligned} \tag{34}$$

The mobile-labor indifference condition (34) pins down the relative price of goods to services:

$$\frac{p^k}{p^c} = \frac{(1-n)^{\alpha-1}}{n^{\alpha-1}} = \left(\frac{1-n}{n} \right)^{\alpha-1}.$$

Total home wage income is $W = \int_0^1 w_i^1 \, di + w^2 + w^3 = p^k(1-\alpha)n^\alpha + p^c(1-\alpha)(1-n)^\alpha + w^3$. All income is saved in the dominant asset.

Old agents. Home old agents receive their money holdings plus seigniorage $\tau_t/3$, where the transfer is divided equally among the continuum of type-1 agents, plus the single type-2 and type-3 agents (we normalize so each "type" receives $\tau_t/3$ of total seigniorage, treating the continuum of type-1 workers as one group). They maximize $K^{\phi}c^{1-\phi}$ subject to $p^k K + p^c c = \Omega^i$, where Ω^i is old-age wealth and K is the Dixit-Stiglitz aggregate (32). By symmetry, the price index is $P^K = p^k$.

Home old agents face no currency friction: all their balances are in the dominant asset, which is universally accepted. Their effective purchasing power equals their gross wealth.

Demand for varieties. With symmetric varieties and a common price p^k, home demand for each variety is uniform: $k_i = K$ for all i. Home agents are indifferent between domestically produced and imported units of each variety.

Seigniorage. The total stock of the dominant asset is $M_t = W_t + s\,W_t^*$ (home savings plus aggregate foreign reserve holdings), so seigniorage is $\tau_t = (\mu - 1)(W_{t-1} + s\,W_{t-1}^*)$.

D.6 Representative Foreign Country

By symmetry, every foreign country i faces the same problem. We describe a representative foreign country, dropping the index i.

Young agents. Foreign country i produces variety i and services. Let p^{k*} denote the price of any variety in terms of the dominant asset (by symmetry $p^{k*} = p^k$ under the law of one price). Wages are:

$$
\begin{aligned}
w^{1*} &= p^{k*}(1-\alpha)\, n^{*\alpha}, \\
w^{2*} &= p^{c*}(1-\alpha)\,(1-n^*)^{\alpha}, \\
w^{3*} &= p^{k*}\alpha\, n^{*\,\alpha-1} = p^{c*}\alpha\,(1-n^*)^{\alpha-1}.
\end{aligned}
$$

Total foreign wage income (per country) is $W^* = w^{1*} + w^{2*} + w^{3*}$.

Portfolio problem. Each foreign young agent saves her entire wage. She allocates savings between the dominant asset (amount a) and foreign money (amount $b = w^{i*} - a$) to maximize effective purchasing power $L(R^H a,\, R^F b)$ from the transaction problem described above. In the deterministic steady state, $R^H = 1/\mu$ and $R^F = 1/\hat{\mu}$. The CES first-order condition gives the dominant-asset portfolio share:

$$s \equiv \frac{a}{a+b} = \frac{\left(\frac{\lambda}{1-\lambda}\right)^{\sigma}\left(\frac{\hat{\mu}}{\mu}\right)^{\sigma-1}}{1+\left(\frac{\lambda}{1-\lambda}\right)^{\sigma}\left(\frac{\hat{\mu}}{\mu}\right)^{\sigma-1}}. \tag{35}$$

Three properties: s is increasing in λ (stronger dominant-asset advantage), increasing in $\hat{\mu}/\mu$ when $\sigma > 1$ (higher foreign inflation makes foreign money less attractive), and—crucially—independent of the goods tariff θ. When $\lambda = 1$ or $\hat{\mu}/\mu \to \infty$, $s \to 1$ and we recover the benchmark.

Old agents. A foreign old agent has effective purchasing power L. She maximizes $K^{\phi}c^{1-\phi}$ subject to $p^{k*}K + p^{c*}c = L$, where K is the Dixit-Stiglitz aggregate.

Demand for varieties. With symmetric varieties and a common price, foreign demand is spread uniformly: $k_j = K$ for all j. Since foreign country i is measure zero, essentially *all* of its goods spending goes to other countries' varieties—requiring cross-border settlement. This is the source of the currency friction.

Foreign seigniorage. Foreign old agents receive foreign seigniorage. Using the foreign money-market clearing condition, $\hat{\tau}_t = (1 - 1/\hat{\mu})(1-s)W_t^*$.

D.7 Equilibrium

Definition 4 (Multi-country equilibrium)**.** *A stationary equilibrium consists of labor allocations* $(n, \{\nu_i\}_{i\in[0,1]}, n^*)$*, prices* (p^k, p^c, p^{c*})*, a portfolio share* s*, money supplies* $(M_t, \hat{M}_t)$*, and consumption allocations such that:*

1. ***Home household optimization.*** *Each home young agent saves all income in the dominant asset. Each home old agent maximizes* $K^{\phi}c^{1-\phi}$ *subject to* $p^c c + p^k K = \Omega^i$*.*

2. ***Home mobile labor allocation.*** *Mobile workers are indifferent across all goods industries and services:*

$$p^k \alpha\, \nu_i^{\alpha-1} = p^c \alpha\,(1-n)^{\alpha-1} \quad \textit{for all } i \in [0,1], \qquad n = \int_0^1 \nu_i\, di.$$

   By symmetry $\nu_i = n$ *for all* i*, which gives* $p^k/p^c = [(1-n)/n]^{\alpha-1}$*.*

3. ***Foreign household optimization.*** *Each foreign young agent saves all income, allocating fraction s to the dominant asset and $1-s$ to foreign money, where s maximizes effective purchasing power (33). Each foreign old agent maximizes $K^{\phi}c^{1-\phi}$ subject to $p^{c*}c+p^{k}K=L$.*

4. ***Foreign labor market clearing.*** *In each foreign country, the mobile wage equals the marginal product in both sectors:*

$$w^{3*}=p^{k*}\alpha\, n^{*\,\alpha-1}=p^{c*}\alpha\,(1-n^{*})^{\alpha-1}.$$

5. ***Services market clearing.*** *Services are non-tradable:*

$$\textstyle\sum_i c^{i}=(1-n)^{\alpha}\ \ \textit{(home)},\qquad \sum_i c^{i*}=(1-n^{*})^{\alpha}\ \ \textit{(each foreign country).}$$

6. ***Global goods market clearing.*** *For each variety j, world demand equals world supply. By symmetry, this reduces to a single aggregate condition:*

$$n^{\alpha}+n^{*\alpha}=\phi[\textstyle\sum_i\Omega^{i}+\sum_i\Omega^{i*}]/p^{k},$$

where the left side is world goods output (home n^{α} plus aggregate foreign $n^{*\alpha}$) and the right side is world goods demand.

7. ***Dominant-asset market clearing.*** $\sum_i m^{i}+s\,W^{*}=M_t$.

8. ***Foreign money market clearing.*** $(1-s)\,W^{*}=e_t\hat{M}_t$.

9. ***Government budget constraints.*** $\tau_t=M_t-M_{t-1}$ *(home);* $\hat{\tau}_t=e_t(\hat{M}_t-\hat{M}_{t-1})$ *(foreign).*

D.8 Reduction to the two-money system

The multi-country structure collapses to the two-money model of the benchmark under the symmetry assumptions.

Under symmetry, every foreign country is identical: same n^{*}, same wages, same portfolio share s. All varieties have the same price, so the Dixit-Stiglitz price index is $P^{K}=p^{k}$ and variety-level demands are uniform. Cobb-Douglas upper-tier preferences therefore imply the same aggregate expenditure shares as in the benchmark: fraction ϕ of old-age expenditure is devoted to tradables and fraction $1-\phi$ to services.

The only new object relative to the benchmark is the portfolio share s. In the benchmark, all foreign saving is placed in the dominant asset. Here, only the fraction s is. That means the reserve-demand mechanism is unchanged except for a scaling: every benchmark term that comes from foreign accumulation of the dominant asset is multiplied by s, while the remaining share $1 - s$ is held in the foreign settlement instrument and does not generate a real transfer to the home country.

This delivers the benchmark steady-state system with the reserve-demand term scaled by s. On the foreign side, the service-market condition gives the goods-sector labor share as the autarky allocation ϕ plus the extra tradables production needed to finance dominant-asset accumulation:

$$n^* = \phi + s\,(1-\phi)\,\frac{\mu-1}{\mu}\,, \tag{36}$$

$$\tag{37}$$

Foreign excess tradables output then equals the real transfer required to support those dominant-asset holdings:

$$K_F = s\,\frac{\mu-1}{\mu}\,n^{*\,\alpha-1}\,. \tag{38}$$

Finally, the home side is the same labor-allocation condition as in the benchmark, with that scaled real transfer on the right-hand side:

$$\phi\,n^{\alpha-1} - n^{\alpha} = s\,(1-\phi)\,\frac{\mu-1}{\mu}\,n^{*\,\alpha-1}\,. \tag{39}$$

The portfolio share s is given by (35) and depends on $(\lambda, \sigma, \mu, \hat{\mu})$. This is the precise sense in which the multi-country model reduces to the two-money benchmark under symmetry: the structure of the equilibrium system is unchanged, and the only modification is that foreign reserve demand is scaled by the endogenous portfolio share s.

What the multi-country structure adds. The multi-country environment does not change the equilibrium allocations, but it provides three things the two-country model cannot:

1. A *reason* for foreign countries to trade with each other: each produces one variety but consumes all, so cross-border goods trade is essential.

2. A *micro-foundation* for reserve-currency demand: the multi-market shopping problem arises from the variety structure—each cross-border purchase is a transaction that may require the reserve currency for settlement.

3. A *structural interpretation* of λ and σ: λ is the effective mass of cross-border goods transactions settled in the reserve currency, determined by invoicing conventions and payment-

network effects; σ reflects the dispersion of settlement requirements across markets.

D.9 Tariff policy

We now introduce a tariff in the multi-country environment and show that tariff neutrality survives.

Tariff. The home government imposes an ad-valorem import tariff $\theta \geq 0$ on all goods varieties imported from foreign countries. The tariff creates a wedge between the price paid by home consumers and the price received by foreign producers. Let p^{k*} denote the producer price of any variety (common across foreign countries by symmetry). Home consumers pay $p^k = (1+\theta)\, p^{k*}$ for imported varieties.

Since home also produces all varieties, the domestic price of all goods—whether home-produced or imported—is $p^k = (1+\theta)\, p^{k*}$ (the tariff-inclusive price, which domestic producers also receive by the law of one price within the home market).

Tariff revenue is $T = \theta\, p^{k*}\, K_F$, where K_F is total imports (aggregate foreign goods absorbed by home). Revenue is rebated lump-sum to home old agents.

Home allocation under the tariff. The home mobile-labor condition continues to equate marginal products across sectors at *domestic* prices:

$$p^k\, \alpha\, n^{\alpha-1} = p^c\, \alpha\, (1-n)^{\alpha-1},$$

which pins down $p^k/p^c = [(1-n)/n]^{\alpha-1}$ as before. The tariff does not enter this condition: home producers face the tariff-inclusive goods price p^k in both sectors.

Home old agents have wealth $\Omega^i = w^i_{t-1} + \tau_t/3 + T/3$ (including tariff revenue). They spend fraction ϕ on goods (at price p^k) and $1-\phi$ on services (at price p^c).

Foreign allocation under the tariff. Foreign producers sell at price $p^{k*} = p^k/(1+\theta)$. But foreign agents do not pay the tariff: they buy foreign varieties at producer prices p^{k*} (cross-border trade among foreign countries is not subject to the home tariff). Their wages, portfolio allocation, and consumption decisions are unchanged:

- The portfolio share s depends on $(\lambda, \sigma, \mu, \hat{\mu})$ and does not contain θ.
- Foreign service-market clearing depends on s, μ, $\hat{\mu}$, and ϕ, not on θ.
- The foreign labor allocation n^* is independent of θ.

Home service-market clearing. Let Ω denote total home old-age wealth. Because tariff revenue is rebated lump-sum,

$$\Omega = M_t + T, \qquad T = \theta p^{k*} K_F,$$

where M_t is the total stock of the dominant asset held by home and foreign agents. Home expenditure shares imply

$$p^c(1-n)^\alpha = (1-\phi)\Omega, \tag{40}$$

$$p^k\left(n^\alpha + K_F\right) = \phi\Omega, \tag{41}$$

where $p^k = (1+\theta)p^{k*}$ is the domestic goods price and $n^\alpha + K_F$ is total home absorption of tradables. Eliminating Ω between (40) and (41) gives

$$n^\alpha + K_F = \frac{\phi}{1-\phi}\frac{p^c}{p^k}(1-n)^\alpha.$$

Using the home labor-market condition $p^k/p^c = [(1-n)/n]^{\alpha-1}$, equivalently $p^c/p^k = (n/(1-n))^{\alpha-1}$, this reduces to

$$\phi n^{\alpha-1} - n^\alpha = (1-\phi)K_F. \tag{42}$$

Hence the home labor allocation depends on the tariff only through the import quantity K_F.

Proposition 9 (Tariff neutrality in the multi-country environment). *For any ad-valorem import tariff $\theta \geq 0$ with revenue rebated lump-sum to home old agents:*

(i) The portfolio share s is independent of θ.

(ii) The foreign labor allocation n^* is independent of θ.

(iii) The home labor allocation n is independent of θ.

(iv) The aggregate import quantity K_F is independent of θ.

Proof. The argument has three steps.

Step 1: s is independent of θ. The portfolio share is determined by the CES first-order condition (33), which depends on asset returns $(R^H, R^F) = (1/\mu, 1/\hat{\mu})$ and the transaction-structure parameters (λ, σ). The tariff θ creates a wedge in goods prices but does not alter any asset return. Therefore s does not depend on θ.

Step 2: n^* and K_F are independent of θ. By the reduction to the two-money system in (36)–(38),

$$n^* = \phi + s(1-\phi)\frac{\mu-1}{\mu}, \qquad K_F = s\frac{\mu-1}{\mu}n^{*\,\alpha-1}.$$

Both objects depend on the tariff only through s, and Step 1 shows that s is independent of θ. Hence both n^* and K_F are independent of the tariff.

Step 3: n is independent of θ. The home-side equilibrium condition is (42),

$$\phi n^{\alpha-1} - n^{\alpha} = (1-\phi)K_F.$$

The left-hand side depends only on n, while the right-hand side depends on the tariff only through K_F. By Step 2, K_F is independent of θ. Therefore the equation determining n is independent of the tariff, so the home labor allocation is independent of θ as well.

The intuition is the same as in the benchmark. The tariff is a goods-price wedge. It cannot alter the foreign portfolio share s, the foreign labor allocation n^*, or the net real transfer K_F, because those are pinned down by the foreign demand for the reserve asset. Once K_F is fixed, the home goods-market and service-market conditions imply the same labor allocation as in the no-tariff economy. □

Remark (Tariff and the Dixit-Stiglitz structure). The tariff applies to home imports of all foreign varieties. With symmetric varieties and a common price, it does not distort the composition of imports—home still demands each foreign variety equally. The tariff contracts the total volume of imports *only if* it changes the net real transfer K_F. Since K_F is pinned by the reserve-demand channel, the tariff does not contract import volume. The measured trade deficit $\theta\, p^{k*} K_F/(p^k(n^\alpha + K_F))$ changes mechanically through the price wedge (a valuation effect), but the real quantity K_F does not.

D.10 Financial tariff

While the goods tariff is neutral, a tax on foreign reserve holdings operates on the asset-return margin and therefore *does* change allocations.

Setup. Suppose the home government imposes a tax $\tau_f \in [0,1)$ on foreign holdings of the dominant asset. The effective real return on the dominant asset for foreign agents becomes $R^H = (1-\tau_f)/\mu$ instead of $1/\mu$. The return on foreign money $R^F = 1/\hat{\mu}$ is unchanged.

Modified portfolio share. Substituting the after-tax return into the CES first-order condition, the portfolio share becomes:

$$s(\tau_f) = \frac{\left(\frac{\lambda}{1-\lambda}\right)^{\sigma} \left(\frac{\hat{\mu}(1-\tau_f)}{\mu}\right)^{\sigma-1}}{1 + \left(\frac{\lambda}{1-\lambda}\right)^{\sigma} \left(\frac{\hat{\mu}(1-\tau_f)}{\mu}\right)^{\sigma-1}}. \tag{43}$$

For $\sigma > 1$, s is strictly decreasing in τ_f: the financial tax makes the dominant asset less attractive, shifting foreign portfolios toward foreign money.

Effect on allocations. Since the steady-state conditions are:

$$n^* = \phi + s\,(1-\phi)\,\frac{\mu-1}{\mu}\,, \tag{44}$$

$$\phi\, n^{\alpha-1} - n^{\alpha} = s\,(1-\phi)\,\frac{\mu-1}{\mu}\, n^{*\,\alpha-1}\,, \tag{45}$$

$$K_F = s\,\frac{\mu-1}{\mu}\, n^{*\,\alpha-1}\,, \tag{46}$$

a reduction in s propagates through all three conditions.

Proposition 10 (Financial tariff effectiveness)**.** *For $\sigma > 1$, an increase in the reserve-accumulation tax τ_f:*

(i) Reduces the portfolio share: $\partial s/\partial \tau_f < 0$.

(ii) Reduces foreign goods-sector labor: $\partial n^*/\partial \tau_f < 0$.

(iii) Increases home goods-sector labor: $\partial n/\partial \tau_f > 0$.

(iv) Reduces home imports: $\partial K_F/\partial \tau_f < 0$.

Proof. Part (i): In (43), $(1-\tau_f)$ enters with exponent $\sigma - 1 > 0$ in the numerator. Increasing τ_f reduces the numerator relative to the denominator.

Part (ii): From (44), n^* is increasing in s. Lower s gives lower n^*.

Part (iv): From (46), K_F depends on s directly and through n^*. Both channels reduce K_F when s falls.

Part (iii): The right-hand side of (45) falls when s falls (both because s enters directly and because n^* falls). The left-hand side $\phi n^{\alpha-1} - n^{\alpha}$ is decreasing in n for $n < \phi$. A smaller right-hand side therefore requires a larger n. □

Interpretation. The financial tariff succeeds where the goods tariff fails. It reduces the trade deficit and reverses deindustrialization. The mechanism is direct: by taxing the return on the reserve asset, the home government makes foreign money relatively more attractive, reducing foreign demand for the dominant asset (s falls). With less foreign savings flowing into the home asset, fewer goods need to flow in the opposite direction, and the net real transfer K_F shrinks. On the production side, the home tradables sector expands (n rises) because the exorbitant-privilege drag is reduced.

Contrast with goods tariff. The goods tariff θ and the financial tariff τ_f target different margins:

Policy instrument	Affects allocations?	Operates on
Import tariff θ	No	Goods-price margin
Financial tariff τ_f	Yes	Asset-return margin
Monetary policy μ	Yes	Asset-return margin

The trade deficit in a reserve-currency economy is pinned by foreign demand for the home asset. Policies that change the attractiveness of the home asset (financial taxes, monetary policy) can alter the trade deficit; policies that change goods prices (tariffs) cannot. This orthogonality is not an artifact of the benchmark's single-currency assumption—it survives in the multi-country environment with differentiated goods, a foreign settlement currency, and endogenous portfolio choice.

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